Effect of delamination on stiffened and un-stiffened shell-A comparative study

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ABSTRACT
In this present paper, the finite element modeling of the shell and the stiffener is presented, wherein multipoint constraint algorithm has been incorporated to model delamination in the shell. This algorithm ensures the compatibility of deformation, equilibrium of resultant forces and moments at delamination crack tip. The undelaminated region is modeled as a single layer of element and the delaminated region as two layers of elements, whose interface contains delamination. The panel is modeled employing eight noded isoparametric shell element and three noded isoparametric beam element for the shell and stiffener, respectively. The compatibility between shell and stiffener is achieved by transforming the degrees of freedom of the stiffener element to the degrees of freedom of the shell element considering eccentricity and curvature. The effect of delamination on the free vibration and transient response due to low velocity impact of the shell with and without stiffener is investigated.

Keywords: Stiffened shell, Delamination, Hertzian contact, Vibration, Impact

1. INTRODUCTION
A twisted cantilever composite cylindrical shell with low aspect ratio can be idealized as turbine blade. The turbine blades are mainly made of thin shells and very often subjected to flutter during operation. To reduce flutter in turbine blades, a rib like structures called stiffeners are added to the thin shells at proper orientation. In many situations, these composite structures are subjected to impact and other type of loadings, which are expected to occur during manufacturing, maintenance and service operation. This may leads to develop debonding between the interface layers of the laminated shell called delamination, which is the most feared damage mode. The presence of delamination reduces the bending stiffness and stability of the structure while the addition of stiffener to the delaminated shell improves the structural stiffness and strength, thereby increases the dynamic characteristics of the structure.

A number of early works have been carried out to study the free vibration response of the laminated stiffened plates and shells. Researchers\cite{1-4} studied the free vibration characteristics of laminated stiffened shells or plates using finite element method. Some notable researchers

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such as Naganarayana et al.\textsuperscript{5}, Yetman et al.\textsuperscript{6}, Milazzo and Oliveri\textsuperscript{7} and Riccio et al.\textsuperscript{8} investigated the delaminated stiffened panels. The low-velocity impact analysis and prediction of impact-induced damage of composite stiffened plate based on layer wise/solid element method and progressive failure model was reported out by Li et al.\textsuperscript{9} while Gong and Lam\textsuperscript{10} reported an approximate solution technique to find the impact response of stiffened composite plate and compared the results with simulated results of LS-DYNA3D. Faggiani and Falzon\textsuperscript{11} presented an intralaminar damage model, which was implemented in ABAQUS and used in a detail finite element model to obtain the response of the stiffened composite panel of carbon fiber as a result of the low-velocity impact. Sun et al.\textsuperscript{12} used finite element and mode superposition method to investigate the problems of uncertainty in the impact force identification of a composite stiffened panel.

Literature review reveals that plenty of literature is available in the theme of free vibration and low velocity impact of stiffened shells but the attention of the researchers has not been focused on twisted delaminated stiffened shell. Hence, the present work aims at investigating the effect of delamination on free vibration and low velocity impact response of laminated twisted unstiffened and stiffened shell. An eight noded isoparametric shell element with five degrees of freedom per node comprising of three translations and two rotations is employed for finite element modeling the cylindrical shell based on first-order shear deformation theory. Multipoint constraints algorithm\textsuperscript{13} has been incorporated in the formulation for modeling delamination at the desired location in the shell. The isoparametric curved three-node beam elements with four degrees of freedom (axial, transverse displacement, flexural and torsional rotation) per node are chosen for modeling of the stiffeners. The compatibility between the shell and stiffener elements is ensured by transforming the nodal degrees of freedom of the stiffener to the shell degrees of freedom considering the curvature and eccentricity of the stiffener. The natural frequencies are evaluated from the standard eigenvalue problem and are solved by QR iteration algorithm. The of contact force due to low velocity impact is computed by using modified Hertzian contact law\textsuperscript{14}.

2. THEORETICAL FORMULATION
The relation between the curvature of twist, twist angle and length of the pretwisted cylindrical shell (shown in Fig. 1) is given as

\[ \tan \phi = -\frac{L}{R_{xy}} \]  

(1)
The generalized strain components composed of the mid plane strains and curvatures of the shell are
\[
\{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}\}^T = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0, \gamma_{xz}^0, \gamma_{yz}^0\}^T + z\{\kappa_x, \kappa_y, \kappa_{xy}, \kappa_{xz}, \kappa_{yz}\}^T
\]  \hspace{1cm} (2)

The constitutive equation for the shell is given by
\[
\begin{bmatrix}
N \\
M \\
Q
\end{bmatrix} =
\begin{bmatrix}
A_{ij} & B_{ij} & 0 \\
B_{ij} & D_{ij} & 0 \\
0 & 0 & S_{ij}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_p \\
\kappa_y \\
\gamma_s
\end{bmatrix}
\]  \hspace{1cm} (3)

where \(A_{ij}, B_{ij}, D_{ij}\) and \(S_{ij}\) are the in-plane, bending-inplane coupling, bending, and transverse shear stiffnesses, respectively. \(\varepsilon_p\), \(\kappa_y\) and \(\gamma_s\) are the in-plane strains, the slopes of the rotations and the transverse shear strains, respectively. For delamination modeling, multipoint constraint algorithm reported by Gim\textsuperscript{13} is incorporated in this formulation.

Figure 1. A typical twisted laminated composite stiffened plate.

An eight noded isoparametric quadratic plate element with five degrees of freedom \((u, v, w, \alpha\) and \(\beta)\) per node is employed for modeling the cylindrical shell. The stiffness and mass matrices of the shell element are determined using standard procedure of finite element method\textsuperscript{15} and given by
\[
[K_{she}] = \int_0^1 \int_0^1 \begin{bmatrix} B \end{bmatrix}^T [D] \begin{bmatrix} B \end{bmatrix} \mu \, d\xi d\eta
\]  \hspace{1cm} (4)
\[
[M_{she}] = \int_0^1 \int_0^1 \begin{bmatrix} N \end{bmatrix}^T \begin{bmatrix} m \end{bmatrix} \begin{bmatrix} N \end{bmatrix} \mu \, d\xi d\eta
\]  \hspace{1cm} (5)

where, \([B],[D],[N]\) and \([m]\) are the strain displacement matrix, elasticity matrix, shape function matrix and general inertia matrix per unit area, respectively.

The isoparametric curved three-noded beam element with four degrees of freedom (axial, transverse displacement, flexural and torsional rotation) per node is chosen for modeling of the stiffener. The shape functions of the x- directional stiffener are expressed as follows:
\[
N_i^x = 0.5\xi_i^x(1 + \xi_i^x) \quad \text{for } i=1,3
\]  \hspace{1cm} (6)
The strain components of the x-directional stiffeners considered are

\[
\begin{bmatrix}
\varepsilon_x^{sx} \\
\gamma_y^{sx} \\
\gamma_z^{sx}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial U^{sx}}{\partial x} + \frac{\partial V^{sx}}{\partial y} + \frac{\partial W^{sx}}{\partial z} \\
\frac{\partial U^{sx}}{\partial y} + \frac{\partial V^{sx}}{\partial z} \\
\frac{\partial U^{sx}}{\partial z}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial u^{sx}}{\partial x} + z \frac{\partial \alpha^{sx}}{\partial x} \\
\frac{\partial u^{sx}}{\partial y} \\
\frac{\partial u^{sx}}{\partial z} - y \frac{\partial \beta^{sx}}{\partial x}
\end{bmatrix}
\]

(7)

The stress resultants of the x-directional stiffener obtained from the stresses developed in the cross-section are given as

\[
\begin{bmatrix}
N_x^{sx} \\
M_x^{sx} \\
T_x^{sx} \\
Q_x^{sx}
\end{bmatrix} = \begin{bmatrix}
\int_{y}^{h} \int_{x}^{h} \sigma_x^{sx} dy dx \\
\int_{y}^{h} \int_{x}^{h} \tau_{yx}^{sx} dy dx \\
\int_{y}^{h} \int_{x}^{h} \tau_{zx}^{sx} dy dx \\
\int_{y}^{h} \int_{x}^{h} \tau_{zx}^{sx} dy dx
\end{bmatrix}
\]

(9)

Rearranging the terms the above equation can be expressed as

\[
\begin{bmatrix}
N_x^{sx} \\
M_x^{sx} \\
T_x^{sx} \\
Q_x^{sx}
\end{bmatrix} = \begin{bmatrix}
A_{11}^{sx} b_{st} + B_{11}^{sx} b_{st} + B_{16}^{sx} b_{st} + B_{16}^{sx} b_{st} \\
B_{16}^{sx} b_{st} + D_{11}^{sx} b_{st} + D_{16}^{sx} b_{st} + D_{16}^{sx} b_{st} \\
D_{16}^{sx} b_{st} + D_{16}^{sx} b_{st} + \frac{1}{6} \left( \bar{Q}_{66}^{sx} + \bar{Q}_{44}^{sx} \right) b_{st} + D_{16}^{sx} b_{st} \\
0 + 0 + 0 + k_s A_{44}^{sx} b_{st}
\end{bmatrix} \begin{bmatrix}
\frac{\partial u^{sx}}{\partial x} \\
\frac{\partial u^{sx}}{\partial y} \\
\frac{\partial u^{sx}}{\partial z} + \frac{\partial \alpha^{sx}}{\partial x} \\
\frac{\partial u^{sx}}{\partial z} - y \frac{\partial \beta^{sx}}{\partial x}
\end{bmatrix}
\]

(10)

Eq. (10) can be expressed in compact form as

\[
\{ F_x^{sx} \} = \{ E_x^{sx} \} \{ \varepsilon_x^{sx} \}
\]

(11)

The element stiffness and mass matrices of the x-stiffener are

\[
[K_{sx}] = \int_{-1}^{1} [T_{sx}]^\top [B_{sx}]^\top [E_{sx}] [B_{sx}] [T_{sx}]^\top [J_{sx}] d\xi
\]

(12)

\[
[M_{sx}] = \int_{-1}^{1} [T_{sx}]^\top [N_{sx}]^\top [m_{sx}] [N_{sx}] [T_{sx}]^\top [J_{sx}] d\xi
\]

(13)

where \([B_{sx}]\) and \([m_{sx}]\) are the strain displacement and inertia matrices of x-directional stiffener. \([T_{sx}]\) is the transformation matrix used to transform the nodal degrees of freedom of the stiffener element to the degrees of freedom of the shell element considering eccentricity of the stiffener i.e. \(e_{sx} = (h+a_d)/2\) .
Similarly the stiffness and mass matrices of the y-directional stiffeners can be obtained. Further, the effect of both eccentricity and curvature will be taken into account for y-stiffeners.

The element stiffness and mass matrices of the stiffened shell element are obtained by adding those of the shell, x-directional stiffener and y-directional stiffener elements as follows:

\[
[K_x] = [K_{she}] + [K_{sx}] + [K_{sy}]
\]  
\[
[M_x] = [M_{she}] + [M_{sx}] + [M_{sy}]
\]

The global stiffness \([K]\) and mass \([M]\) matrices are obtained by assembling the element stiffness and mass matrices, respectively.

The generalized dynamic equation is derived from Lagrange's equation of motion is expressed as

\[
[M]\{\ddot{\delta}\} + [K]\{\delta\} = 0
\]

where, \([M]\), \([K]\) and \{\delta\} are global mass matrix, elastic stiffness matrix and displacement vector, respectively.

The natural frequencies are evaluated from the standard eigenvalue problem and are solved by QR iteration algorithm\(^{15}\).

\[
[A]\{\delta\} = \lambda\{\delta\}
\]

where, \([A] = [K]^{-1}[M]\) and \(\lambda = 1/\omega_n^2\)

In case of low velocity impact, it is assumed that the vibration of the hard impactor can be neglected. The contact forces based on modified Hertzian contact law during loading, unloading and reloading cycle are determined as\(^{14}\)

\[
F_C = k\alpha^{1.5}
\]

\(0 < \alpha \leq \alpha_m\) loading, \(\alpha_m\) is the maximum indentation and \(\alpha_0\) is the permanent indentation in a loading/unloading cycle. The contact stiffness \(k\) of the cylindrical shell is defined by

\[
F_c = F_m \left[ \frac{\alpha - \alpha_0}{\alpha_m - \alpha_0} \right]^{2.5}
\]

\(F_m\) is the maximum contact force, \(\alpha_m\) is the maximum indentation and \(\alpha_0\) is the permanent indentation in a loading/unloading cycle. The contact stiffness \(k\) of the cylindrical shell is defined by

\[
F_c = F_m \left[ \frac{\alpha - \alpha_0}{\alpha_m - \alpha_0} \right]^{1.5}
\]
\[ k = \frac{4}{3} \left[ \frac{1}{r_i} + \frac{1}{2R_y} \right]^{-1/2} \]

where \( r_i, v_i \) and \( E_i \) are the radius, Poisson's ratio and modulus of elasticity of the spherical impactor. \( E_2 \) is the modulus of elasticity of the cylindrical shell transverse to fibre direction.

The local indentation at the contact point is determined as

\[ \alpha(t) = w_i - w_j \cos \phi \]  \hspace{1cm} (23)

where \( w_i \) and \( w_j \) are the displacements of the mid-plane of the cylindrical shell and the spherical impactor at the impact point in the direction of impact.

The contact force vector obtained during impact between the shell and impactor is given by

\[ \{ F \} = \begin{bmatrix} 0 & 0 & \ldots & F_{ci} & \ldots & 0 & 0 \end{bmatrix}^T \]  \hspace{1cm} (24)

The governing dynamic equilibrium equation derived using Lagrange's equation of motion in global form of the stiffened shell at time \((t + \Delta t)\), neglecting effects of damping is expressed as

\[ \begin{bmatrix} M \end{bmatrix} \ddot{\delta}^{t+\Delta t} + \begin{bmatrix} K \end{bmatrix} \delta^{t+\Delta t} = \{ F \}^{t+\Delta t} \] \hspace{1cm} (25)

The equation of motion of the rigid impactor is given by

\[ m_j \ddot{w}_i + F_C = 0 \] \hspace{1cm} (26)

The solution of the Eqs. (25) and (26) are obtained by Newmark's time integration algorithm by selecting a suitable time step.

3. RESULTS AND DISCUSSION

The converged mesh size of 8 x 8 is considered for the entire analysis. Table 1 shows the agreement of the present result with Nayak and Bandyopadhyay\(^3\) and Das and Chakravorty\(^{17}\), which comprises natural frequencies of antisymmetric cross-ply crossed stiffened plate. Table 2 presents the comparative study of the non-dimensional fundamental frequencies of twisted composite plates with different fiber orientation previously reported by Qatu and Leissa\(^{18}\). The accuracy of delamination modeling is furnished in Table 3 wherein the problem considered by Parhi et al.\(^{19}\) and Acharyya et al.\(^{20}\) is resolved. The comparison of time histories of contact force and in-plane stress of a fully clamped laminated plate impacted at the center by a striker with velocity 22.6 m/s previously investigated by Chun and Lam\(^{21}\) is presented with a converged time step of 2 \(\mu\)s in Fig. 2 to support the validation of the code in the respect of low velocity of impact. The ability of the present formulation in respect of stiffener, twist angle, delamination and low velocity impact is well established. Thus, it is
obvious that the model can effectively investigate free vibration and low velocity impact problems of composite delaminated stiffened shell with pretwist.

**Table 1.** Natural frequencies (Hz) of simply supported antisymmetric cross-ply($0°/90°$) crossed stiffened plate with "eccentric at bottom" stiffeners($0°/90°$).

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Nayak and Bandyopadhyay$^3$</th>
<th>Das and Chakravorty$^{17}$</th>
<th>Present FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1141.00</td>
<td>1123.17</td>
<td>1142.03</td>
</tr>
<tr>
<td>2</td>
<td>2394.17</td>
<td>2367.77</td>
<td>2398.12</td>
</tr>
<tr>
<td>3</td>
<td>2415.82</td>
<td>2407.57</td>
<td>2417.10</td>
</tr>
<tr>
<td>4</td>
<td>2646.18</td>
<td>2656.00</td>
<td>2646.31</td>
</tr>
</tbody>
</table>

The geometric and material properties of the graphite-epoxy composite stiffened shell considered for the entire analysis are as follows:

$L=0.4$ m, $b=0.2$ m, $R_y=0.1$ m, $h=0.02$ m, $b_{st}=h/10$, $d_{st}=2h$, $E_1=144.8$ GPa, $E_2=9.65$ GPa, $G_{12}=G_{13}=4.14$ GPa, $G_{23}=3.45$ GPa, $v_{12}=0.30$, $\rho=1389.23$ kg/m$^3$

**Table 2.** Non-dimensional fundamental frequencies of three layer $[\theta/-\theta/\theta]$ graphite-epoxy twisted plates. $L/b=1$, $b/h=20$, Twist angle( $\phi$ )=$30°$

<table>
<thead>
<tr>
<th>$\theta$ (Deg.)</th>
<th>Qatu and Leissa$^{18}$</th>
<th>Present FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9553</td>
<td>0.9431</td>
</tr>
<tr>
<td>15</td>
<td>0.8759</td>
<td>0.8629</td>
</tr>
<tr>
<td>45</td>
<td>0.4831</td>
<td>0.4752</td>
</tr>
<tr>
<td>75</td>
<td>0.2582</td>
<td>0.2572</td>
</tr>
<tr>
<td>90</td>
<td>0.2434</td>
<td>0.2431</td>
</tr>
</tbody>
</table>

The entire analysis is based on the composite shell with stiffener placed symmetrically along x- and y-direction. The stacking sequence of the plate and the stiffener are same. The delamination crack front always stretches across the full width of the shell. The boundary condition considered for the cantilevered stiffened shell is given below:

At $x=0$, $u=v=w=\alpha=\beta=0$

**Table 3.** Fundamental frequencies (Hz) of composite ($[0°/90°]_2$) cylindrical shells for different extents of centrally located mid-surface delamination for simply supported boundary conditions.

<table>
<thead>
<tr>
<th>R/a</th>
<th>C/a</th>
<th>Parhi et al.$^{19}$</th>
<th>Acharyya et al.$^{20}$</th>
<th>Present FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0</td>
<td>129.04</td>
<td>128.99</td>
<td>129.03</td>
</tr>
<tr>
<td></td>
<td>0.5 (25%)</td>
<td>104.56</td>
<td>104.51</td>
<td>104.55</td>
</tr>
<tr>
<td></td>
<td>0.75 (56.25%)</td>
<td>98.24</td>
<td>98.19</td>
<td>98.24</td>
</tr>
<tr>
<td>10.0</td>
<td>0</td>
<td>103.03</td>
<td>103.04</td>
<td>103.02</td>
</tr>
<tr>
<td></td>
<td>0.5 (25%)</td>
<td>69.60</td>
<td>69.61</td>
<td>69.60</td>
</tr>
<tr>
<td></td>
<td>0.75 (56.25%)</td>
<td>59.88</td>
<td>59.92</td>
<td>59.92</td>
</tr>
</tbody>
</table>

The stiffened shell is always assumed to be impacted at the center by a spherical impactor of radius 5 mm with an initial velocity of 3 m/s. The value of mass density, Poisson’s ratio and Young’s modulus of the spherical impactor are 7960 kg/m$^3$, 0.3 and 210 GPa, respectively.
3.1 Effect of delamination on free vibration response

Fig. 3 shows the variation of non-dimensional fundamental frequency (NDFF, $\sigma = \omega_n L^2 \sqrt{\rho / E_h^2}$) of twisted ($\phi = 15^\circ$) laminated ([45/-45/45/-45/45]$_s$) unstiffened shell, x-directional stiffened shell ($n_x = 1$), y-directional stiffened shell ($n_y = 1$) and crossed stiffened shell ($n_x = n_y = 1$) with percentage of delamination. Delamination is considered at the mid-plane of the ten layered angle ply composite shell. Maximum fundamental frequency is obtained in the crossed stiffened shell while minimum fundamental frequency is observed in the unstiffened shell. It is evident that increase in percentage of delamination reduces the value of NDFF in all cases, as normally expected. Comparing x- and y- directional stiffeners, it is observed that y- directional stiffeners are found to be more efficient in increasing the NDFF than x- directional stiffeners. Fig. 4 shows the variation of NDFF with respect to percentage of delamination of the unstiffened and stiffened shell corresponding to single (ND=1) and multiple delaminations (ND=3). In case of multiple delaminations (ND=3), the delaminations are considered at the mid-plane but at three different locations across the thickness. The first delamination is in between the 3rd and 4th ply, second delamination is in between 5th and 6th ply and the last one is in between 7th and 8th ply, respectively. It reveals that there is further decrease in NDFF due to multiple delaminations. The percentage decrease in NDFF due to 25%, 50% and 75% single and multiple delaminations are illustrated in Fig. 5. It reveals that maximum percentage decrease in NDFF corresponding to ND=1 and ND=3 is observed in y-directional stiffened shell while minimum percentage reduction in NDFF is found in x-directional stiffened shells.
3.2 Effect of delamination on transient response under low velocity impact

The effect of single and multiple delaminations on transient response of the twisted ($\phi = 15^\circ$) composite ($[45/-45/45/-45/45]$) unstiffened and stiffened shell ($n_y=1$) subjected to low velocity impact are illustrated in Fig. 6, wherein the contact force history, shell displacement history and history of in-plane stresses are presented. In the case of stiffened shell, a single y-directional stiffener ($n_y=1$) is considered because it renders maximum stiffness to the structure as observed from the free vibration analysis. The mid-plane delamination of 75% is considered in both stiffened and unstiffened shell having a single (ND=1) and multiple delaminations (ND=3). The spherical steel ball is impacted at the center of the twisted stiffened shell. The in-plane stresses ($\sigma_x$, $\sigma_y$) are computed at centre of the top surface of the
laminated composite stiffened shell i.e. \((L/2, 0, -h/2)\). It is observed that the contact force increases with addition of stiffener while maximum contact force is obtained in the undelaminated stiffened shell. It is evident that delamination in both stiffened and unstiffened shell reduces the magnitude of contact force while increases the shell displacement. The maximum shell displacement and minimum contact force is observed corresponding to ND=3 in both stiffened and unstiffened shell. The in-plane stress, \(\sigma_x\) of the undelaminated shell and the stiffened shell is found to be tensile in nature while in delaminated cases, it becomes compressive. Maximum compressive value of \(\sigma_x\) is observed in the unstiffened shell with multiple delaminations (ND=3). The magnitude of \(\sigma_y\) is observed tensile in nature except the unstiffened shell with ND=3 wherein compressive stress is noticed because of maximum reduction in structural stiffness.

![Graphs](image)

**Figure 6.** Effect of delamination on histories of contact force, shell displacement and inplane stresses at the impact point. ND: Number of delamination, US: Unstiffened shell, SS: Stiffened Shell.

### 4. CONCLUSION

The finite element model of the delaminated stiffened shell is presented and the accuracy of the formulation is well established with results available in the literature. The major conclusions drawn from the parametric studies are summarized as follows:
The presence of delamination in both stiffened and unstiffened shell decreases the structural stiffness thereby reduces the fundamental frequency. The effect of delamination is found minimum in x-directional stiffened shell as compared to unstiffened and y-directional stiffened shell. It is concluded from the transient response of the delaminated panel that delamination reduces the value of contact force and increases the shell displacement and it has also striking effects on the magnitude of in-plane stresses wherein a change in the nature of in-plane stresses is also noticed.

REFERENCES