Hygro-thermo mechanically- induced fracture analysis of edge crack Piezomagnetic laminated composite plate using DCM

Khushbu Jain\textsuperscript{a}, Achchhe Lal\textsuperscript{b}
\textsuperscript{a}Research Scholar, Department of Mechanical Engineering, SVNIT, Surat-7, India
\textsuperscript{b}Parul Institute of Technology, Parul University, Waghodia Vadodara- India
\textsuperscript{c}Department of Mechanical Engineering, SVNIT, Surat-7, India

ABSTRACT

This paper deals with Hygro-thermo- mechanically induced fracture analysis of edge crack symmetric angle ply Piezomagnetic laminated composite plate for a uniaxial tensile load. The finite element model combined with thermo-mechanically induced micromechanical model is used to solve the governing equation with basic formulation based on higher order shear deformation plate theory (HSDT) using displacement correlation method (DCM) based on isoquater point element characteristic length. Parametric studies are carried out to examine the effect of stacking sequences, piezoelectric and magnetostrictive layers, and temperature and moisture content conditions with normalized stress intensity factor.

Keywords: Stress Intensity Factor, Displacement correlation method, Piezomagnetic laminated composite plate

1. INTRODUCTION

Composite materials have led to use the fibre reinforced materials in aircraft, automotive industries and various other engineering applications due to superior mechanical properties compared to conventional materials. During applications, this structure is subjected to very high intensity of hygro-thermo-mechanical loadings. Therefore the analysis of thermal loading & piezoelectric effects is also extremely important for the structural integrity and stability of the structures. Various work has been done by researcher in this direction, Ke et al.[1] presented the near tip strain as a ductile fracture criterion and compared with both crack surface opening displacement and J-integral criteria. Chandra et al. (1987)[2] presented a finite element technique to compute the SIF through the J- integral for patched, unpatched, edge cracked and centre-cracked plates Ju (2010) et al. [3] investigated SIFs of a sharp V-notch formed from several anisotropic materials by using image-correlation experiments, to evaluate the displacement field and H-integral Kaman [4] investigated fracture toughness of single edge notched fiber reinforced composite plates experimentally and carried numerical analysis by utilizing ANSYS finite element package software Toledo et al.[5] carried out the calibration of a numerical micro

* Send correspondence to
Khushbu Jain: E-mail: jain17khushbu@gmail.com, 7201080968
Achchhe Lal: E-mail: achchhelal@medsvn.ac.in, 09824442503
mechanical model for general composite materials and simulated the behaviour of each component by a general elastoplastic anisotropic model. Panda et al. [6] carried out the three-dimensional finite element investigation of mixed-mode interlaminar fracture behavior in thermomechanically loaded FRP laminated composites by using Modified crack closure integral methods. Upadhyay et al. [7] presented an analytical solution for the nonlinear flexural response of elastically supported cross-ply and angle-ply laminated composite plates under hygrothermal environment. Zenkour et al. [8] carried out the investigation of deflections and stresses of the multilayered angle-ply composite plates subjected to temperature and moisture effects by introducing the sinusoidal shear deformation plate theory. Zenkour et al. [9] extended the sinusoidal shear deformation plate theory to study the static response of laminated plates resting on elastic foundations subjected to the general hygro-thermal stresses. Hadjiloizi et al.[10-11] developed comprehensive micromechanical model for the analysis of a smart composite piezo-magneto-thermoelastic thin plate with rapidly-varying thickness is developed in the present paper Jun Lei et al. [12] determine fracture parameters of interfacial cracks in transverse isotropic magneto-electro-elastic composites; a displacement extrapolation formula was derived. Rajan et al. [13] investigated Shape control and free vibration analysis of piezolaminated plates subjected to electro mechanical loading are evaluated using finite element method.

It is evident from the available literature that , evaluation of NSIF and its effect on the crack propagation under such loading conditions using micromechanics approach is rarely available in the literature.The fracture analyses of piezomagneto composite laminates with Hygro-thermo mechanically- induced edge cracks are very complex and need more research. In the present analysis, an attempt is made to address the problem

2. MATHEMATICAL FORMULATION

The numerical analysis of a rectangular piezo magneto laminated composite plate is subjected to constant uniform tensile load at one edge, with the bottom edge clamped. The plate is meshed with, isoparametric quad eight noded elements as shown in Figure 1(a). The isoparametric quarter point elements with characteristic length $\Delta b$ of the element as explained in [4] are located around the crack tip as shown in Figure 1(b), so that the singularity of stress and strain fields can be modelled accurately.

2.1 Displacement correlation method (DCM)

The DCM is one of the simplest methods used to evaluate the SIFs. It consists of correlating numerical results of displacements using sliding relative displacement (CSRd) at specific locations on the crack surfaces. In the isoparametric quarter point finite element model, at the crack tip, the crack opening displacement (COD) and the crack sliding displacement (CSD) are given by Kaman[4] & Kim[14]
\[ \text{CSRD}^2 = \text{COD}^2 + \text{CSD}^2 \]  

(1)

For quarter point singular elements as shown in Figure 1(b), pure mode-I SIFs can be estimated as follows using the COD and CSD as [4,14]

\[ K_I = \frac{1}{4} \sqrt{\frac{2\pi}{\Delta b}} \left( D(4\Delta u_{34} - \Delta u_{56}) - B(4\Delta v_{34} - \Delta v_{56}) \right) \frac{AD - BC}{AD - BC} \]  

(2)

where, \( \Delta u_{34}, \Delta u_{56} \) and \( \Delta v_{34}, \Delta v_{56} \) are the relative displacements at the crack tip in the x- and y-directions at locations (3,4) and (5,6), \( r \) is the distance from the crack tip along the x direction, and \( \Delta b \) is a characteristic length associated with the crack tip elements. \( A, B, C \) and \( D \) are given by

\[ A = R_x \left[ \frac{i}{\mu_1 - \mu_2} (\mu_1 p_2 - \mu_2 p_1) \right], \quad B = R_x \left[ \frac{i}{\mu_1 - \mu_2} (p_2 - p_1) \right] \]

\[ C = R_x \left[ \frac{i}{\mu_1 - \mu_2} (\mu_1 q_2 - \mu_2 q_1) \right], \quad D = R_x \left[ \frac{i}{\mu_1 - \mu_2} (q_2 - q_1) \right] \]

(3)

Displacements at the crack tip along x- and y-directions can be written as [4].

\[ u = K_I \sqrt{\frac{2\pi}{\pi}} R_x \left[ \frac{i}{\mu_1 - \mu_2} (\mu_1 q_2 - \mu_2 q_1) \right] + K_{II} \sqrt{\frac{2\pi}{\pi}} R_x \left[ \frac{i}{\mu_1 - \mu_2} (q_2 - q_1) \right] \]

(4)

\[ v = K_I \sqrt{\frac{2\pi}{\pi}} R_x \left[ \frac{i}{\mu_1 - \mu_2} (\mu_1 p_2 - \mu_2 p_1) \right] + K_{II} \sqrt{\frac{2\pi}{\pi}} R_x \left[ \frac{i}{\mu_1 - \mu_2} (p_2 - p_1) \right] \]

(5)
Where the parameters $\mu_1$ and $\mu_2$ are the eigen value of field equation with positive imaginary part for composite plate and compliance coefficients can be obtained from Kaman[4].

$$s_{11}\mu^4 + (2s_{12} + s_{66})\mu^2 + s_{22} = 0 \quad (6)$$

$$\overline{S}_{ij} = \begin{pmatrix}
\frac{1}{E_x} & -\nu_{xy} & \frac{\nu_{xy}}{E_y} & 0 \\
-\nu_{xy} & \frac{1}{E_y} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & \frac{1}{G_{xy}}
\end{pmatrix} \quad (7)$$

Where $E_x$, $E_y$, $\nu_{XY}$, $G_{XY}$ are the young moduli in x- and y- directions, poissons’ ratio and shear modulus, respectively. To perform a numerical analysis, the finite element analysis software, ANSYS and MATLAB[R2010a] finite element (FE) code is used. The homogenous plane element (Plane82) with eight nodes and having seven DOFs at each node are taken for computation of results. The material properties are expressed and calculated by using plane stress strain equation.

The parameters $p_k$ and $q_k$ ($k = 1, 2$) are given by

$$p_k = S_{11}\mu_k^2 - S_{16}\mu + S_{12}, \quad q_k = S_{12}\mu_k^2 + \frac{S_{22}}{S_{26}} - S_{26} \quad (8)$$

### 2.2 Displacement field model

In the present study as explained earlier, to calculate numerical values of displacement of the plate and crack face, the Reddy’s higher order shear deformation theory using $C^1$ continuity is transformed into $C^0$ continuity by assuming derivatives of out-of-plane displacement as separate degree of freedom (DOFs). The modified displacement field with $C^0$ continuity along x, y and z directions for arbitrary laminated composite plate are now written as [14]

$$\bar{u} = u + f_1(z)\phi_1 + f_2(z)\theta_1, \quad \bar{v} = v + f_3(z)\phi_2 + f_4(z)\theta_2, \quad \bar{w} = w \quad (9)$$

Where $\bar{u}, \bar{v}$ and $\bar{w}$ denote the displacements of a point along (x-y-z) coordinate axis, $u, v, w$ are corresponding displacement of a point on the mid plane $\phi_1$ and $\theta_2$ are the rotations at $z=0$ of normal to mid surface with respect to y and x axis respectively. The parameters $\theta_1(=w_{xx})$ and $\theta_2(=w_{yy})$ are the slopes along x, y axes respectively where ($\phi$) denotes partial differential equation. In the Eq. (9), the function $f_1(z)$ and $f_2(z)$ can be expressed as [14]

$$f_1(z) = C_1z - C_2 z^3 \quad \text{and} \quad f_2(z) = -C_4 z^3 \quad \text{with} \quad C_1 = 1; C_2 = C_4 = 4/3h^3 \quad (10)$$

The displacement field vector for the modified $C^0$ continuous model is denoted as

$$q = (u \quad v \quad w \quad \theta_2 \quad \theta_1 \quad \phi_2 \quad \phi_1) \quad (11)$$
The linear strain for the crack structure considered here, the relevant strain vector consisting of strains in terms of mid-plane deformation, rotation of normal and higher order terms associated are written as

$$\{ \varepsilon \} = \{ \varepsilon_1 \ \varepsilon_2 \ \varepsilon_3 \ \varepsilon_4 \ \varepsilon_5 \} = [T] \{ \overline{\varepsilon} \}$$  \hspace{1cm} (12)

Where \( T = \)

$$\begin{bmatrix}
1 & 0 & 0 & Z & 0 & 0 & Z^3 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & Z & 0 & 0 & Z^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & Z & 0 & 0 & Z^3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & Z^2 \end{bmatrix}$$  \hspace{1cm} (13)

$$\{ \varepsilon \} = \{ \varepsilon_1^0 \ \varepsilon_2^0 \ \varepsilon_3^0 \ \varepsilon_4^0 \ \varepsilon_5^0 \}$$  \hspace{1cm} (14)

The strain energy of piezomagnetic laminated composite plate is written as [17]

$$U_m = \frac{1}{2} \int_V \{ \varepsilon \}^T \{ \sigma \} dV = \frac{1}{2} \int_\Lambda (\{ \varepsilon \}^T \{ \sigma \}) dV$$

$$= \frac{1}{2} \int_\Lambda (\{ \varepsilon \}^T \{ D \} \{ \varepsilon \}) dA - \frac{1}{2} \int_\Lambda \{ \varepsilon \}^T \{ D_p \} dA - \frac{1}{2} \int_\Lambda \{ \varepsilon \}^T \{ D_m \} dA$$  \hspace{1cm} (15)

where, \( D \) is the laminate elastic stiffness matrix for piezo-magnetic layers [19]

The parameter \( \{ \varepsilon \} \) is defined as the relevant strain vector consisting of linear strain (in terms of mid plane deformation, rotation of normal and higher order terms), thermal and piezo strains vectors associated with the displacements are expressed as

$$\{ \varepsilon \} = \{ \varepsilon^L \} - \{ \varepsilon^N \} - \{ \varepsilon^P \} - \{ \varepsilon^M \}$$  \hspace{1cm} (16)

The parameter \( \{ \varepsilon \} \) is the strain vector consisting of linear \( \{ \varepsilon^L \} \) strains respectively. The linear strain tensor \( \{ \varepsilon^L \} \) for the plate element using HSDT can be rewritten as

$$\{ \varepsilon^L \} = [B] \{ q \}$$  \hspace{1cm} (17)

where \([B]\) and \( \{ q \} \) are the geometrical matrix and displacement field vector. The electric field vector \( \{ \varepsilon^E \} \) as given in Eq. (15) can be represented which is neglected as Shegokar and Lal [19]

$$\{ \varepsilon^E \} = \frac{V_p}{h_p} \{ d_{31} \}$$  \hspace{1cm} (18)

Where, \( d_{31} \) is the piezoelectric strain constants in x direction which can be evaluated from the electric coefficients in the longitudinal directions of the layer using transformation matrix. Here \( V_p \) and \( h_p \) are the applied voltage to the actuators in the thickness direction and thickness of piezoelectric layer, respectively.

The magnetic field vector \( \{ \varepsilon^M \} \) as given in Eq. (15) can be represented which is neglected in present analysis as Shegokar and Lal [19]
\[ \{ \varepsilon^M \} = \frac{B_M}{h_M} \{ \chi_{31} \} \]  

(19)

Where \( \chi_{31} \) is the magnetostrictive strain constant in \( x \)-direction which can be evaluated from the magnetic coefficients in the longitudinal direction of the layer using transformation matrix. Here \( B_M \) and \( h_M \) are the applied magnetic field to the actuators in the thickness direction and thickness of magnetic layer, respectively.

From Eq. (15), the electric field displacement vector \( \{ D_p \} \) and can be written as

\[ \{ D_p \} = [e]^T \{ e \} + [k]\{ E_z \} \]  

(20)

The piezoelectric constant matrix \( [e] \) is given by,

\[ [e] = \begin{bmatrix} 0 & e_{11} \\ e_{31} & 0 \end{bmatrix} \]  

(21)

here \( [k] \) is the dielectric displacement coefficient matrix and defined as

\[ [k] = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{33} \end{bmatrix} \]  

(22)

The electric field vector \( \{ E_z \} \) can be defined as

\[ \{ E_z \} = - \left( \frac{\partial \phi}{\partial z} \right) \]  

(23)

where \( \phi \) is the electric potential and can be defined as

\[ \phi(x, y, z) = \phi^{(0)}(x, y) + z\phi^{(1)}(x, y) + z^2\phi^{(2)}(x, y) \]  

(24)

Substituting Eq. (22) in Eq. (23), the electric field vector \( \{ E_z \} \) can be further written as

\[ \{ E_z \} = -\left( \phi^{(1)} + 2z\phi^{(2)} \right) = -(N_1\phi_U + 2zN_2\phi_U) \]  

(25)

where, \( \phi_L \) and \( \phi_U \) are the electric potential corresponding to lower and upper piezoelectric layers, respectively. The parameter \( N_1 \) and \( N_2 \) are shape functions respectively. From Eq. (15), the magnetic field displacement vector \( \{ D_m \} \) and can be written as

\[ \{ D_m \} = [s]^T \{ e \} + [\chi]\{ M_z \} \]  

(26)

The magnetic constant matrix \( [s] \) can be expressed as

\[ [s] = \begin{bmatrix} 0 & s_{11} \\ s_{31} & 0 \end{bmatrix} \]  

(27)

here \( [\chi] \) is the magnetostrictive displacement coefficient matrix and defined as

\[ [\chi] = \begin{bmatrix} \chi_{11} & 0 \\ 0 & \chi_{33} \end{bmatrix} \]  

(28)

The magnetic field vector \( \{ M_z \} \) can be defined as

\[ \{ M_z \} = - \left[ \frac{\partial \psi}{\partial z} \right] \]  

(29)

where \( \psi \) is the magnetic potential and can be defined as
\[ \psi(x, y, z) = \psi^{(0)}(x, y) + z\psi^{(1)}(x, y) + z^2\psi^{(2)}(x, y). \]  

Substituting Eq. (21) in Eq. (22), the electric field vector \([E]\) can be further written as

\[ \{M_e\} = -\left(\psi^{(0)} + 2z\psi^{(2)}\right) = -\left(N_L\psi_U + 2zN_L\psi_U\right) \]

where, \(\psi_L\) and \(\psi_U\) are the magnetic potential corresponding to lower and upper magnetostrictive layers, respectively.

Substituting Eq. (16), (22), and (26), in Eq. (15) once obtained as strain energy and can be represented as linear \((U_{bl})\)

\[ U_{bi} = \frac{1}{2} \int_v \left( (\varepsilon^T) \left[ Q(\varepsilon_i) - eE - sM \right] - E^T \left[ e^T(\varepsilon_i) + kE \right] - M^T \left[ s^T(\varepsilon_i) + \chi M \right] \right) dV \]

From Eq. (32), the electric field displacement and magnetic field displacement components are written as,

\[ \{E\} = [T_e]\{E^0\} \quad \{M\} = [T_m]\{M^0\} \]

the linear potential energy with differential operator.

\[ U_{bi} = \frac{1}{2} \int_A \left( A^T L^T D L A - A^T L^T D_{1e} L_{e2} \phi - \phi^T L_{e1} D_{e1} L_{e2} \phi - \phi^T L_{e1} D_{e2} \phi \right) \right) dA \]

Where in Eq.(32) the linear strain vector can be rewritten as

\[ \{\varepsilon\} = [L]\{\Lambda\} \]

Where \(L\) is the linear strain differential operator and the piezoelectric strain vector and the magnetic strain can be further written as

\[ \{E^0\} = [L_E]\{\phi\} \quad \{M^{(0)}\} = [L_m]\{\psi\} \]

The parameter \([L_\psi]\) is a differential operator of piezoelectric and magnetic layers.

In the present paper eight noded isoparametric elements with seven degrees of freedom per node is employed for finite element plate modelling. For this type of element, the displacement, electric potential and magnetic potential vectors and the element geometry can be expressed

\[ \{\Lambda\}^{(e)} = \sum_{i=1}^{NN} [N_i]\{q_i\}^{(e)}; \quad \{\phi\} = \sum_{i=1}^{NN} [N_i]\{\phi_i\}^{(e)}; \quad \{\psi\} = \sum_{i=1}^{NN} [N]_x\{\psi_i\}^{(e)}, x = \sum_{i=1}^{NN} N_i x_i, y = \sum_{i=1}^{NN} N_i y_i; \]

where \(N_i, \{\Lambda\}, \{\phi\}, \text{ and } \{\psi\} \) are the interpolation function for the \(i\)th node and the vector of unknown displacements, electric potential and magnetic potential vectors for the \(i\)th node, \(NN\) is the number of nodes per element and \(x_i\) is the Cartesian coordinate of the \(i\)th node.

Adopting Gauss quadrature integration numerical magnetostrictive stiffness matrix, respectively and displacement and dielectric load vectors, respectively can be obtained by transforming expression in \(x\) coordinate system to natural coordinate system \(\xi\).

The governing equation of motion can be derived using Hamilton principle, which is generalization of the principle of virtual displacement.
\[
\frac{\partial (U_{bl})}{\partial q} - \frac{\partial (U_2 + U_3)}{\partial q} = 0
\]  

(38)

Where \( U_3 \) Work done due external applied load can be written as

\[
U_3 = \sum_{e=1}^{NE} \{q\}^{(e)T} \{F\}^{(e)}
\]

(39)

Where \( F^{(e)} \) is elemental force vector given by \( F^{(e)} = (0 \ q(y) \ 0 \ 0 \ 0 \ 0)^T \)

The potential energy \( (U_2) \) storage due to thermo magneto loadings can be written as

\[
U_2 = \frac{1}{2} \int_\Omega N_{STMHE} (w, \zeta) \ d\Omega
\]

(40)

\[
U_2 = \sum_{e=1}^{NE} U_2^{(e)} = \frac{1}{2} \sum_{e=1}^{NE} \{q\}^{T(e)} \lambda \left[ K_e^{(e)} \right] \{q\}^{(e)}
\]

(41)

where, \( \lambda \) and \( \left[ K_e \right] \) are defined as the thermo-magneto-piezoelectric buckling load parameters and the global geometric stiffness matrix (arises due to thermo-piezoelectric loadings), respectively.

\[
\left[ K_e \right]^{(e)} = \int_{A(e)} B^{(e)T} N_{STMHE} B^{(e)} \ dA
\]

(42)

Where, \( N_{STMHE} \) is the pre buckling thermo electro magneto stresses resultant per unit length and can be expressed as

\[
N_{STM} = N_0^T + N_0^p + N_0^M
\]

(43)

The parameters \( B_e = \begin{bmatrix} 0 & \frac{dN_0}{dx} & 0 & 0 & \frac{dN_0}{dx} & 0 & 0 \end{bmatrix} \)

By substituting Eqs (39),(41) in Eq (38), ones obtain equation for the element subjected transverse load \( F \) can be expressed as

\[
\begin{bmatrix}
K_s & K_{1PL} & K_{1ML} \\
K_{1PL} & K_{2PL} & 0 \\
K_{1ML} & 0 & K_{2ML}
\end{bmatrix} - \lambda \begin{bmatrix}
K_s & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\phi \\
\psi
\end{bmatrix} = \begin{bmatrix}
F
\end{bmatrix}
\]

(44)

After elimination of \( \phi \) and \( \psi \), Eq. (44) can be written as

\[
\begin{bmatrix}
K
\end{bmatrix}^{*} \begin{bmatrix}
q
\end{bmatrix} = \begin{bmatrix}
F
\end{bmatrix}
\]

(45)

Where, \( K^{*} = K_{L}^{(e)} - \lambda [K_{e}]^{(e)} \)

(46)

Where \( [K] \{q\} \{F\} \) are global stiffness matrixes, nodal displacement, and force vector. The nodal displacement \( \{q\} \) in can be computed from equation Eq (45) substituting nodal displacement in Eq (11) the displacement field vectors \( u \) and \( v \) with all degree of freedom. By using displace field
vectors. SIF given in Eq (2) is computed. In the deterministic case the solution of Eq (2) can be obtained using standard solution procedure.

3. RESULTS AND DISCUSSION

To show the accuracy and efficiency of the present method, validation, and conversion study is carried out by developing a computer program in MATLAB [R2012a] environment with utilizing ANSYS-13 software. For comparison purpose the present results are compared with literature. The finite element model combined with hygro-thermo-mechanically induced micromechanical model is used to solve the governing equation with basic formulation based on higher order shear deformation plate theory (HSDT) using displacement correlation method (DCM) for Piezomagnetic laminated composite plate. The dimensions of the plate used in this study unless otherwise stated are length \((a) = 203.2\) mm, width \((w) = 18\) mm, total thickness \((t) = 1.235\) mm with crack length \((c_l) = 9\) mm and crack opening \((c_o) = 4\) mm as shown in Fig.4.1 (a) and explained by Kaman [4].

The plate is subjected to uniform tensile loading acting on the top edge, with the bottom edge clamped. The material property for piezo magneto laminated composite plate used in the present analysis shown in Table 1 unless otherwise stated.

The following analytical and normalized values mode I SIF are used in present analysis

\[ f(a/w) = 1.12 - 0.231(a/w) + 10.55(b/w)^2 - 21.72(b/w)^3 + 30.39(b/w)^4 \]

\[ K_I = \frac{K_{Jf}}{\sigma \sqrt{\pi a}}, \text{for tensile stress} \]

Where, are the dimensional values of first mode SIF. The parameters \(\sigma\) and \(a\) are represented as uniaxial tension and crack length respectively. Parametric studies are carried out to examine the effect of stacking sequences, temperature, moisture, piezoelectric and magnetostrictive layers.

3.1 Validation of fracture toughness (\(K_{JC}\))

The validation of the present method is carried out. In this set the fracture toughness \((K_{JC})\) values are obtained by combining DCM with HSDT for a rectangular single edge V-notched \([0^0\theta^0\theta^0\theta^0]\) symmetric angle ply laminated composite plate with \((\theta = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ)\) and are compared with the experimental fracture toughness \(K_{JC}\) values available in the literature [4]. It is observed that the present results obtained by MATLAB program based on DCM combined with HSDT are in excellent agreement with the results of experimental study presented by Kaman [4]. Therefore, for further computation of numerical results, MATLAB program based on HSDT is used. And shown in Table 1
3.2 Validation of Isotropic edge crack Plate with unit load

In this example, the method proposed in the present study is applied to the finite plate of isotropic material having material parameters as $E = 72.4$ GPa and $v = 0.3$ with a edge crack of length $a$ is considered. A plate with height $H (=254 \text{ mm})$, width $W (=127 \text{ mm})$ and thickness $(t =1.27 \text{ mm})$ and is subjected to uniaxial tensile load ($\Delta T = 25^\circ \text{c}$, $\Delta C = 0\%$) is considered. The validation for the present study has been done with different crack length w.r.t to SIF for isotropic edge crack plate with unit load shown in Fig 2.

![Validation of present study for Unit load](image)

**Fig 2** Validation of Isotropic edge crack Plate with unit load

3.3 Parametric Study of evaluating the normalized NSIFs of the Piezo-magneto laminated composite plate.

In the present investigation firstly the mechanical properties of the unidirectional angle ply lamina composite plate subjected to hygro-thermal effects. The effect of variation of crack length with the position of piezomagnetic layers on the normalized mean of $K_I$ of an edge cracked laminated piezomagnetic composite plate subjected to mechanically applied tensile loading with laminated angles A,B,C,D,E,F ($\theta =15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$),It is observed that with the increase of crack length, the NSIF increases. As the position of piezoelectric and magnetostrictive layers applied from top to bottom of fibres layers, the NSIF increases.shown in Table 3.

Then in the further study, the NSIFs $K_I$ of the through thickness single edge cracked symmetric angle ply [0$/$$\phi$/0] laminated composite plate with $\phi$ ($=15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$) subjected to uniaxily acting tensile, shear and combined (tensile and shear) mechanical stresses along with hygro-thermo effects are evaluated for finding the effect of variation of moisture concentration, temperature, volume fraction of fibres, crack angle, crack length and applied stress on the fracture response in terms of NSIFs.
The following material properties of the fibre and matrix material as given by [18] and [40] are used for the computation of NSIFs throughout this study (unless stated otherwise). $E_{f1}=230\text{GPa}$, $E_{f2}=13.79\text{GPa}$, $E_m=3.45\text{GPa}$, $G_{f12}=9.0\text{GPa}$, $v_{f1}=0.203$, $v_m=0.35, a_f=0.99\times10^{-6}/\text{C}, a_{f2}=10.08\times10^{-6}/\text{C}, a_m=72.0\times10^{-6}/\text{C}, \beta_m=0.33, T_{go}=216^\circ\text{C}$

Tables 5 examines the effect of variation of $\Delta T$ and $\Delta C$ respectively on the NSIFs of the different laminated composite plates subjected to hygro-thermo mechanically applied tensile, stress. The analysis is carried out for $\Delta T$ (=50°, 100°, 150° and 180° c) with $\Delta C$ (=0, 0.05, 0.10 and 0.20%) with $\Delta T$ (=100° c) by keeping $V_f$ (=0.5, 0.6).

The values of NSIFs obtained for variation of $\Delta T$ and $\Delta C$ follow a similar trend of variation with position of piezoelectric and magnetostrictive layers applied from top to bottom of fibres layers. The values of NSIFs obtained for change in $\Delta T$ are slightly higher than those obtained for change in $\Delta C$. The results obtained indicate that, with the increase in the temperature and moisture the values of calculated NSIFs $K_1$ increases and ultimately increases the fracture resistance of the composite plates, due to the effect of energy release/propagation rate. And with the increase in moisture content the values of calculated NSIFs $K_{I, decrease}$ due to the effect fibre become weak and which decreases the fracture toughness.

**Table 1** Mechanical properties of piezo magneto laminated composite plates [4][17]

<table>
<thead>
<tr>
<th>Carbon fibre reinforced composite plates</th>
<th>PZT-5</th>
<th>Terfenol-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal modulus $E_{11}$ (MPa)</td>
<td>1.729 e5</td>
<td>E(GPa)</td>
</tr>
<tr>
<td>Transverse modulus $E_{22}$ (MPa)</td>
<td>1.078 e4</td>
<td>$\alpha$ (1/K)</td>
</tr>
<tr>
<td>Shear modulus $G_{12}$ (MPa)</td>
<td>3.638 e3</td>
<td>$\rho$ (Kg/m$^3$)</td>
</tr>
<tr>
<td>Poissos Ratio</td>
<td>0.33</td>
<td>$\nu$</td>
</tr>
<tr>
<td>$k_p$ (W/mK)</td>
<td>2.1</td>
<td>$k_m$ (W/mK)</td>
</tr>
<tr>
<td>$d_{31}$(m/V)</td>
<td>2.54e-10</td>
<td>$\chi_{31}$(F/m)</td>
</tr>
</tbody>
</table>

**Table 2** Comparison of critical stress intensity factor ($K/Ic$) for different lamination schemes of laminated composite plates with present DCM

| $\{0^\circ/15^\circ/15^\circ/10^\circ\}$ | 4404.62 | 4620.1 |
| $\{0^\circ/30^\circ/30^\circ/10^\circ\}$ | 3478.88 | 3485.6 |
| $\{0^\circ/45^\circ/45^\circ/10^\circ\}$ | 3192.76 | 3256.9 |
| $\{0^\circ/60^\circ/60^\circ/10^\circ\}$ | 3535.14 | 3551.4 |
| $\{0^\circ/75^\circ/75^\circ/10^\circ\}$ | 2835.29 | 2895.4 |
| $\{0^\circ/90^\circ/90^\circ/10^\circ\}$ | 2488.43 | 2536.1 |
Table 3 Effect of variation of crack length with position of piezomagnetic layers on the normalized mean of $KI$ of an edge cracked laminated piezomagneto composite plate subjected to mechanically applied tensile loading.

<table>
<thead>
<tr>
<th>$a/w$</th>
<th>Parameter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.38</td>
<td>P=1,m=1</td>
<td>2.3179</td>
<td>2.9865</td>
<td>2.8703</td>
<td>3.3841</td>
<td>2.3825</td>
<td>2.1678</td>
</tr>
<tr>
<td></td>
<td>P=2,m=1</td>
<td>2.8215</td>
<td>3.0711</td>
<td>3.1509</td>
<td>2.9957</td>
<td>2.6901</td>
<td>2.5978</td>
</tr>
<tr>
<td></td>
<td>P=3,m=1</td>
<td>2.6831</td>
<td>3.0552</td>
<td>3.2229</td>
<td>2.9332</td>
<td>2.6058</td>
<td>2.4949</td>
</tr>
<tr>
<td></td>
<td>P=2,m=2</td>
<td>2.4314</td>
<td>2.588</td>
<td>2.6847</td>
<td>2.5012</td>
<td>2.3012</td>
<td>1.4722</td>
</tr>
<tr>
<td>0.5</td>
<td>P=1,m=1</td>
<td>3.2021</td>
<td>-0.6887</td>
<td>-0.3977</td>
<td>3.5059</td>
<td>3.1785</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P=2,m=1</td>
<td>3.5740</td>
<td>4.1078</td>
<td>4.3951</td>
<td>3.4320</td>
<td>3.4236</td>
<td>3.4400</td>
</tr>
<tr>
<td></td>
<td>P=3,m=1</td>
<td>3.5102</td>
<td>3.9345</td>
<td>4.1132</td>
<td>3.7782</td>
<td>3.4609</td>
<td>3.4295</td>
</tr>
<tr>
<td></td>
<td>P=2,m=2</td>
<td>3.1434</td>
<td>3.3306</td>
<td>3.1297</td>
<td>3.1330</td>
<td>3.0137</td>
<td>3.0940</td>
</tr>
</tbody>
</table>

Table 4 Effect of variation of temperature moisture effect of variation of $(\Delta T, \Delta C, V_f)$ with position of piezomagnetic layers on the normalized mean of $KI$ of an edge cracked laminated piezomagneto composite plate with volume fraction

<table>
<thead>
<tr>
<th>Temperature,Moisture</th>
<th>Parameter</th>
<th>Vf=0.5</th>
<th>Vf=0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T=50$, $\Delta C=0$</td>
<td>P=0,m=0</td>
<td>2.7338</td>
<td>2.7186</td>
</tr>
<tr>
<td></td>
<td>P=0,m=1</td>
<td>3.7678</td>
<td>3.543</td>
</tr>
<tr>
<td></td>
<td>P=1,m=0</td>
<td>3.7678</td>
<td>3.543</td>
</tr>
<tr>
<td></td>
<td>P=1,m=1</td>
<td>3.7601</td>
<td>3.543</td>
</tr>
<tr>
<td></td>
<td>P=2,m=1</td>
<td>3.5100</td>
<td>3.4825</td>
</tr>
<tr>
<td></td>
<td>P=3,m=1</td>
<td>3.4500</td>
<td>3.6704</td>
</tr>
<tr>
<td></td>
<td>P=2,m=3</td>
<td>3.2352</td>
<td>3.1861</td>
</tr>
<tr>
<td></td>
<td>P=2,m=2</td>
<td>3.1478</td>
<td>3.1042</td>
</tr>
<tr>
<td></td>
<td>P=3,m=3</td>
<td>3.1478</td>
<td>3.1042</td>
</tr>
<tr>
<td>$\Delta T=100$, $\Delta C=0.05$</td>
<td>P=0,m=0</td>
<td>2.6734</td>
<td>2.6663</td>
</tr>
<tr>
<td></td>
<td>P=0,m=1</td>
<td>3.2326</td>
<td>3.6044</td>
</tr>
<tr>
<td></td>
<td>P=1,m=0</td>
<td>3.2326</td>
<td>3.6044</td>
</tr>
<tr>
<td></td>
<td>P=1,m=1</td>
<td>3.6098</td>
<td>3.2918</td>
</tr>
<tr>
<td></td>
<td>P=3,m=1</td>
<td>3.4990</td>
<td>3.4412</td>
</tr>
<tr>
<td></td>
<td>P=2,m=3</td>
<td>3.2298</td>
<td>3.1810</td>
</tr>
<tr>
<td></td>
<td>P=2,m=2</td>
<td>3.0944</td>
<td>3.1319</td>
</tr>
<tr>
<td></td>
<td>P=3,m=3</td>
<td>3.0944</td>
<td>3.1319</td>
</tr>
<tr>
<td>$\Delta T=150$, $\Delta C=0.10$</td>
<td>P=0,m=0</td>
<td>2.5370</td>
<td>2.5468</td>
</tr>
<tr>
<td></td>
<td>P=0,m=1</td>
<td>3.3944</td>
<td>3.1394</td>
</tr>
<tr>
<td></td>
<td>P=1,m=0</td>
<td>3.3944</td>
<td>3.1394</td>
</tr>
<tr>
<td></td>
<td>P=1,m=1</td>
<td>3.6576</td>
<td>3.6424</td>
</tr>
<tr>
<td></td>
<td>P=3,m=1</td>
<td>3.5112</td>
<td>3.4726</td>
</tr>
<tr>
<td></td>
<td>P=2,m=3</td>
<td>3.2214</td>
<td>3.1726</td>
</tr>
<tr>
<td></td>
<td>P=2,m=2</td>
<td>3.2072</td>
<td>3.1465</td>
</tr>
<tr>
<td></td>
<td>P=3,m=3</td>
<td>3.2072</td>
<td>3.1465</td>
</tr>
<tr>
<td>$\Delta T=180$, $\Delta C=0.20$</td>
<td>P=0,m=0</td>
<td>2.1760</td>
<td>2.2248</td>
</tr>
<tr>
<td></td>
<td>P=0,m=1</td>
<td>3.3524</td>
<td>3.3206</td>
</tr>
<tr>
<td></td>
<td>P=1,m=0</td>
<td>3.3524</td>
<td>3.3206</td>
</tr>
<tr>
<td></td>
<td>P=1,m=1</td>
<td>3.7522</td>
<td>3.5225</td>
</tr>
<tr>
<td></td>
<td>P=3,m=1</td>
<td>3.5221</td>
<td>3.4809</td>
</tr>
<tr>
<td></td>
<td>P=2,m=3</td>
<td>3.2109</td>
<td>3.1614</td>
</tr>
<tr>
<td></td>
<td>P=2,m=2</td>
<td>3.3747</td>
<td>3.0918</td>
</tr>
<tr>
<td></td>
<td>P=3,m=3</td>
<td>3.3747</td>
<td>3.0918</td>
</tr>
</tbody>
</table>

4. CONCLUSION

In this work, the DCM formulation of the hygro-thermo mechanical problems of piezo-magneto laminated composite plate are proposed and investigated. The hygro-thermo mechanical
behavior of piezo-magneto composite plate with various parameters has been studied. As the position of piezoelectric and magnetostrictive layers applied from top to bottom of fibres layers, properties on the NSIFs is determined, with increase in volume fraction NSIF decrease as due to fibre bonding become weak and strength decreases. With temperature and moisture coefficient of thermal expansion get affected due to compressive forces developed in it due to in plane thermo loading. With increase in lamination angle there is increase in NSIF and for same layer position of piezo or magnetic layer there is same effect.

REFERENCES


