Nonlinear Vibration of Pre and Post Buckled Composite Shells under Hygro-thermal Environment

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1. INTRODUCTION & OBJECTIVE

Composite shell structures are especially used in aircraft, defence and space structures. The composite shell structures may undergo variation of thermal and moisture conditions. The displacements, velocities and accelerations in a flight maneuvers resulted from control surface movements depend on the elevation and maneuver of the flight. Due to the variation of temperature [1], pressure and moisture the laminated composite shell components are subjected to internal pressure, thermal and moisture. Especially, dynamic loads act on aerospace structures and exhibit nonlinear deflection behavior. It is important to carry out research on the nonlinear defection study of the shells under hygro-thermo-elastic environmental conditions.

In the present investigation a finite element formulation [2] is presented to analyze the nonlinear analysis of doubly curved laminated composite shells. The nonlinear deflection and frequency of vibration depend on geometry, material data and the environmental loading conditions.

2. NONLINEAR FINITE ELEEMT FORMULATIONS

The laminated doubly curved shell panel used in the present formulation [3] is derived using an orthogonal curvilinear coordinate system, is shown in Fig. 1. α_1 and α_2 are defined along the lines of principal curvatures, and the normal coordinate is ζ. The shell panel dimensions of *a* and *b* are along the α_1 and α_2 directions, respectively.

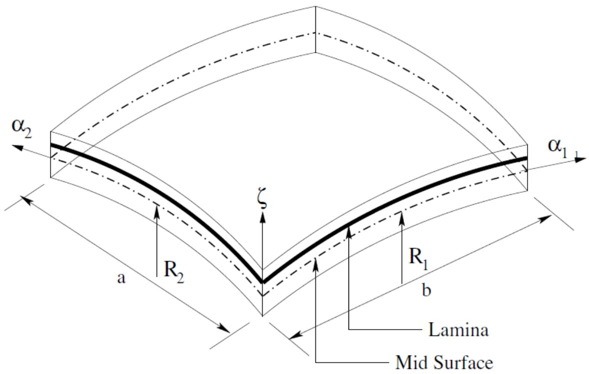

Fig. 1 Doubly curved laminated shell panel

The nonlinear strains [4] are expressed as

$$\begin{Bmatrix} \varepsilon^0_{1nl} \\ \varepsilon^0_{2nl} \\ \varepsilon^0_{12nl} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}\left[\left(\varepsilon^0_1\right)^2 + \left(\varepsilon^0_3\right)^2 + \left(\varepsilon^0_5\right)^2\right] \\ \frac{1}{2}\left[\left(\varepsilon^0_2\right)^2 + \left(\varepsilon^0_4\right)^2 + \left(\varepsilon^0_6\right)^2\right] \\ \left(\varepsilon^0_1\varepsilon^0_4 + \varepsilon^0_2\varepsilon^0_3 + \varepsilon^0_5\varepsilon^0_6\right) \end{Bmatrix} \quad (1)$$

where, the mid-surface strains and curvature terms are explicitly expressed, as follows:

$$\begin{aligned}
\varepsilon_1^0 &= \frac{1}{A_1}\frac{\partial u_1^0}{\partial \alpha_1} + \frac{u_2^0}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{w^0}{R_1} \\
\varepsilon_2^0 &= \frac{1}{A_2}\frac{\partial u_2^0}{\partial \alpha_2} + \frac{u_1^0}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{w^0}{R_2} \\
\varepsilon_3^0 &= \frac{1}{A_1}\frac{\partial u_2^0}{\partial \alpha_1} - \frac{u_1^0}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \\
\varepsilon_4^0 &= \frac{1}{A_2}\frac{\partial u_1^0}{\partial \alpha_2} - \frac{u_2^0}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} \\
\varepsilon_5^0 &= \frac{1}{A_1}\frac{\partial w^0}{\partial \alpha_1} - \frac{u_1^0}{R_1} \\
\varepsilon_6^0 &= \frac{1}{A_2}\frac{\partial w^0}{\partial \alpha_2} - \frac{u_2^0}{R_2}
\end{aligned} \tag{2}$$

$$\begin{aligned}
\kappa_1 &= \frac{1}{A_1}\frac{\partial \theta_1}{\partial \alpha_1} + \frac{\theta_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \\
\kappa_2 &= \frac{1}{A_2}\frac{\partial \theta_2}{\partial \alpha_2} + \frac{\theta_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} \\
\kappa_3 &= \frac{1}{A_1}\frac{\partial \theta_2}{\partial \alpha_1} - \frac{\theta_1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \\
\kappa_4 &= \frac{1}{A_2}\frac{\partial \theta_1}{\partial \alpha_2} - \frac{\theta_2}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}
\end{aligned} \tag{3}$$

The equations of motion are derived using the virtual work principle. The vibration analysis [5] is carried out by solving the generalized eigen value problem.

3. RESULTS AND DISCUSSION

The present results and the published results are in very good agreement. A doubly curved simply supported spherical panel subjected to a point load at the centre is analyzed for the post buckling and vibration analyses. The following material data are used:

E_{11} = 181 GPa, E_{22} = 10.3 GPa, G_{12}= G_{13}= 7.17 GPa and G_{23}= 3.58 GPa and γ = 0.28

Fig. 2 shows the variation of fundamental frequencies with respect to the applied loads. The fundamental frequencies decrease in pre-buckling region and increase in post-buckling region.The non-dimensional natural frequencies at at uniform rise in temperature are shown in Fig. 3.

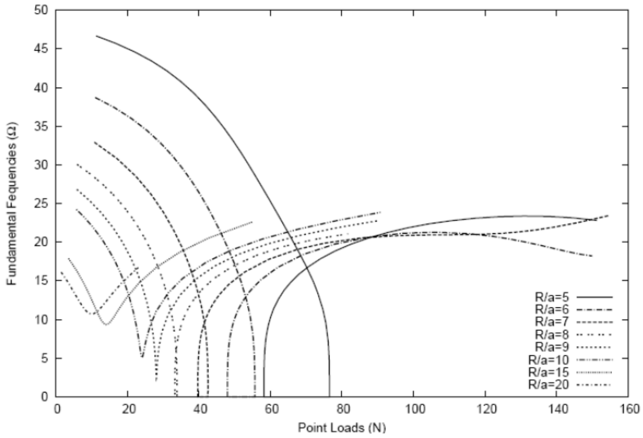

Fig. 2 Effect of shallowness on vibration characteristics of $[0/90]_2$ laminated spherical panels

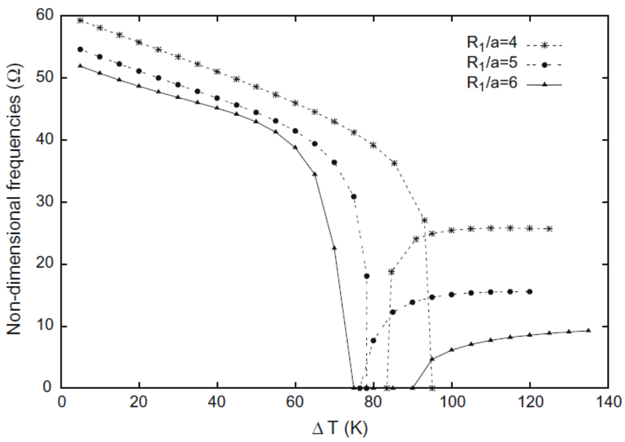

Fig 3 Vibration characteristics of [0/90]s laminated conical panel (a/h=400) due to uniform rise in temperature

The nonlinear static and dynamic hebaviour of laminated composite shell panels are studied under hygro-thermal environmental conditions. Nonlinear finite element is developed for a general shell. The thermal and moisture may induce instability of laminated panels. Pre- and post buckling behavior is studied and the nonlinear dynamic responses are also studied. The nonlinear deflection and frequency of vibration depend on geometry, material data and the environmental loading conditions.

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