Study on Parametric Instability of an Asymmetric Tapered Sandwich Beam Configuration by Computational Method

Das S. Author1[a], Pradhan M. Author2[b*], Dash P. R. Author3[b*]
[a] M.tech Student (MDA)VSSUT,Burla, [b*]Faculty of Mechanical Engg. Department, VSSUT, Burla 768018, India

1. INTRODUCTION & OBJECTIVE

In this research article, the dynamic stability analysis of an asymmetric sandwich beam subjected to a pulsating axial load consisting of both static and dynamic part acting at the centre of the cross-section has been studied for different boundary conditions like clamped-clamped, clamped-pinned and pinned-pinned boundary conditions. In this research work, the configuration of the system is tapered along depth as well as width.

The equation of motion and associated boundary conditions are derived by use of extended Hamilton's principle which are non-dimensionalised later on. By the use of approximate series solution for different boundary conditions taking from the last research work and using Galerkin's method, the matrix equation of motion are obtained. From the matrix equation mass matrix, stiffness matrix and other matrices are obtained. Then by the use of modal matrix, a set of uncoupled Hill's equation have been found out. Putting the acceleration of the system and the non-dimensional dynamic load acting on the centroid of the system as zero, the static buckling load for first three modes have been found out and dynamic analysis of the system has been done by the applications of Saito-Otomi condition[2] to the uncoupled Hill's equation.

The effect of shear parameter, core loss factor, width as well as depth taper parameters on the static stability and zones of parametric instability for the above mentioned three different boundary conditions have been investigated. The effect of above factors are studied and presented through series of graphs by the use of matlab.

The application of sandwich construction in the design of structural elements for aerospace industry is common and widespread. Generally, this type of construction is used where strength required is large; weight required is as much less as possible and having less vibration when it is subjected to external force. Generally sandwich structures with visco-elastic core possess high internal damping with large shear deformation of visco-elastic core having better vibration absorption and noise reduction ability under dynamic conditions. In this case parametric stability analysis of the system has been done. First parametric vibration of a system has been noticed by FARADAY in the year 1831. In our case the system is consisting of three layers. The upper layer and lower layer are elastic layers which will provide strength to the system. The middle layer is visco-elastic layer and it will reduce vibration and simultaneously will reduce the weight of the system as its density is less. Since, most of the load is carried out by the base layer and the upper layer is only meant for constraining the system i.e. why the thickness of the upper layer and lower layer has been taken with different values. Up to now no research work has been done for a system which is tapered along the depth as well as along the width. Since, by considering the taper of the beam both in the width and depth, the material can be saved, strength and stability of the beam can be obtained. Hence, the system chosen by us is tapered both along width and depth.

2. METHODS OF ANALYSIS

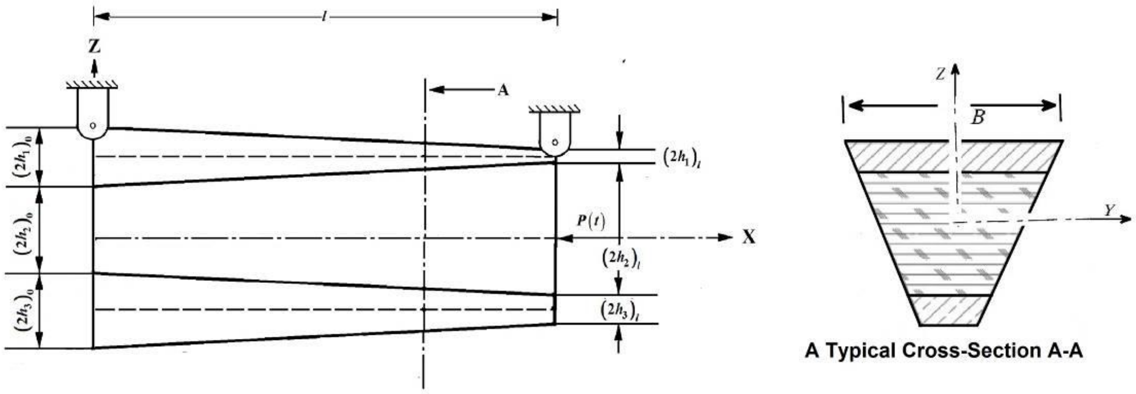

Fig 1: Configuration of the System

The assumptions made for the derivation of equations of motion are same as used for the previous research work done by Pradhan M. and Dash P.R. [2] as given in reference. The non-dimensional equation of motion and boundary conditions have been obtained by Hamilton's Principle and non-dimensional equation of motion is given by

$$\bar{m}\left(\bar{w}_{,\bar{t}\bar{t}}\right)+\left(\bar{P}\bar{w}_{\bar{x}}\right)_{,\bar{x}}+\left(D\bar{w}_{,\bar{x}\bar{x}}\right)_{\bar{x}\bar{x}}-g^{*}Y\{B_1\left(\bar{U}+\bar{w}_{,\bar{x}}\right)\}=0 \quad (1)$$

$$g^{*}YB_1\bar{w}_{,\bar{x}}-Y(K\bar{U}_{,\bar{x}})_{,\bar{x}}+g^{*}YB_1\bar{U}=0 \quad (2)$$

Where

$\bar{m}$: Mass/unit length — $\bar{w}_{,\bar{x}\bar{x}}$: $\dfrac{\delta^2\bar{w}}{\delta\bar{x}^2}$

$\bar{U}(x,t)$: Axial displacement — $\bar{P}$: Non-dimensional amplitude for dynamic loading

$\bar{w}_{,\bar{t}\bar{t}}$: $\dfrac{\partial^2\bar{w}}{\partial\bar{t}^2}$ — $\bar{w}_{\bar{x}}$: $\dfrac{\partial\bar{w}}{\partial\bar{x}}$

Y : Geometric Parameter — D,K : Function of depth taper parameter and Young's Module

g^{*} : $g(1+j\eta)$

B_1 : Width of the beam

By the use of approximate series solution, the matrix equation of motion has been obtained. From the matrix equation, the static buckling load for various parameters has been obtained and regions of parametric instability have been obtained from the uncoupled Hill's equation by Saito-Otomi conditions from the reference[2].

3. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

The effect of parameters like shear parameter, core loss factor, depth as well as width taper parameters have been studied on the static buckling load as well as for the dynamic stability of the system and it is found that the effect of shear parameter, core loss factor on static buckling load remains same as in earlier cases. In addition to the effect of shear parameter and core loss factor, we studied the effect of depth and width taper parameters. With the decrease in depth taper parameter, both static and dynamic the stability increases where as with the increase in width taper parameter, it is found that dynamic stability improves and static stability remains unchanged. So, for better stability, system should be diverging instead of converging along +ve x and +ve z direction. Here, it is found that the combination resonance zones are not appearing. This is due to the fact that the matrix equations are already uncoupled before the application of modal matrix.

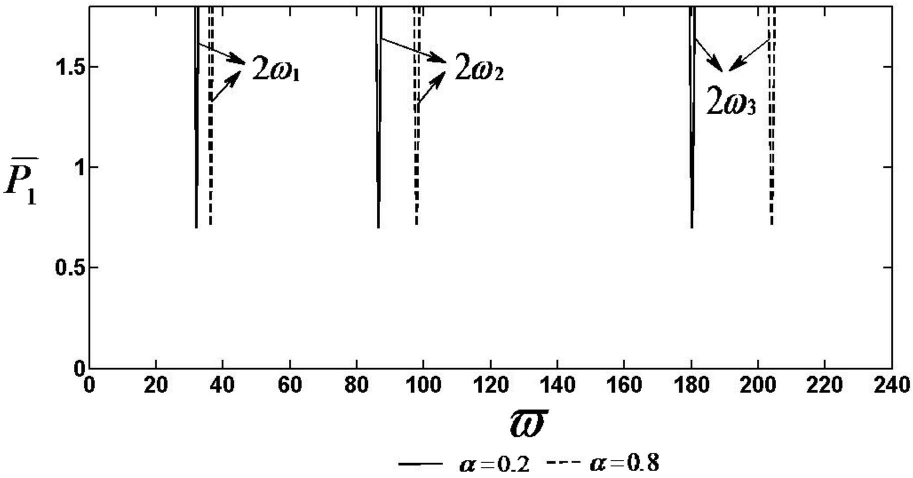

Fig 2: Effect of width taper parameter on regions

Of instability zone in P-P case with

$$\eta = 0.1, \overline{P}_0 = 0.5, \alpha_1 = 0.625, \alpha_3 = 0.4, g = 0.1, Y = 35$$

The above figure shows the effect of variation of width taper parameter on zones of instability with $\alpha = 0.2$ and $\alpha = 0.8$. We can conclude from above figure that with increase in width taper parameter, dynamic stability of the system increases.

In the same way, studies have been done for other parameters and conclusions have been derived.

REFERENCES

1. Faraday, Michael. "On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces." *Philosophical transactions of the Royal Society of London* 121 (1831): 299-340..
2. Pradhan M.,Dash P R:Stability of an asymmetric tapered sandwich beam resting on a variable Pasternak foundation subjected to a pulsating axial load with thermal gradient.*Composite Structures 140 (2016) 816–834*
3. Pradhan M.,Mishra M.K., Dash P R. Stability analysis of an asymmetric tapered sandwich beam with thermal gradient. *Procedia Engineering 144 (2016) 908 – 916*
4. Kar, R. C., and T. Sujata. "Dynamic stability of a tapered symmetric sandwich beam." *Computers & structures* 40.6 (1991): 1441-1449.
5. Ray, K., and R. C. Kar. "Parametric instability of a sandwich beam under various boundary conditions." *Computers & structures* 55.5 (1995): 857-870.