Hand Held FFT ANALYSER FOR MODEL ANALYSIS

Tuhin Bandopadhyay[2], Shashank Gandhi, N. K. Peyada, A. Ghosh[1]
Indian Institute of Technology, Kharagpur
Kharagpur- 700302, India

1. INTRODUCTION

Any component, capable of vibrating, has natural frequencies. Natural frequencies are the 'vibrational' fingerprint of an object. The natural mode is an internal dynamic property of a freely oscillating structure; the consequence of which is particular deformation behaviour. This is so, for any structures as well as for objects of our daily life. Essentially, these natural frequencies are characterized by the geometry and material of an object. As a rule, any oscillating body has several such natural frequencies associated with vibrating modes.

Many techniques have been invented to measure the natural frequencies of a structure experimentally; all following few basic principles:

- Apply vibration to the structure by any kind of impulsive hitting.
- Sense the vibration using any of the contacting or non-contacting sensors, including piezoelectric, magnetic, capacitive/electrostatic, optical etc.
- Determination of natural frequencies, ideally, using a spectrum analyzer which uses Fast Fourier Transform to convert the vibration data from time domain to frequency domain.

Researcher Mr. Rahul D. Mankar and Dr. M.M.Gupta in their paper[1] has reported that there is high scope of low priced hand held FFT analyser. Our present development is related to a low cost hand held device to measure the natural frequencies of any vibrating structure. This device will be useful in Structural Health Monitoring (SHM) including crack determination.

2. BASIC COMPONENTS USED

1. Accelerometer(ADXL345): The ADXL345 is a small, thin, ultralow power, 3-axis accelerometer with high resolution (13-bit) measurement at up to ±16 g. Digital output data is formatted as 16-bit two's complement and is accessible through either a SPI (3 or 4-wire) or I2C digital interface. ADXL345 has 3200Hz output data rate. Hence maximum frequency measurable using this is 1600Hz according to Nyquist sampling theorem[2].
2. Arduino Mega 2560: The Arduino Mega 2560 is a microcontroller board based on the ATmega2560. It has 54 digital input/output pins (of which 15 can be used as PWM outputs), 16 analog inputs, 4 UARTs (hardware serial ports), a 16 MHz crystal oscillator, a USB connection, a power jack, an ICSP header, and a reset button. It contains everything needed to support a microcontroller like accelerometers.
3. LCD Display: A QAPASS 1602 LCD Display is used to display the output results.

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* Further Author Information:
[1] Email : anup@aero.iitkgp.ernet.in, anupghoshtab@gmail.com
[2] Email : banerjeetuhin801@gmail.com

3. WORKING PRINCIPLE

Using the accelerometer, the acceleration of the vibrating structure is captured and stored through Arduino programming. First Fourier Transform (FFT) of the stored data samples are calculated using Arduino FFT library and the amplitude (y-axis) of the transformed data points are plotted against frequency (x-axis). The corresponding x-points of the certain high peaks of the curve denote the natural frequencies of the structure.

3.1 FFT: A FFT algorithm computes the discrete Fourier transform (DFT) of a sequence. DFT of a sequence is given by:

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn} \ \ where\ W_N = e^{-j2\pi/N}\ and\ 0 < k < N-1 \ \dots\dots\dots\dots\dots.(1)$$

N = total no of data samples and x(n) are the data points for n = 1(1)N Fourier analysis converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa. An FFT rapidly computes such transformations by factorizing the DFT matrix into a product of sparse (mostly zero) factors. As a result, it manages to reduce the complexity of computing the DFT from $O(N^2)$, which arises if one simply applies the definition of DFT (Equation (1)), to O(N*log N) where N is the data size. There are several algorithms available for computing FFT. Cooley-Tukey Radix 2 algorithm[3] is used in the Arduino program to calculate FFT . According to this algorithm, let us consider $N = 2^n$ data points and split them into two $N/2$-point data sequences $f_1(n)$ and $f_2(n)$, corresponding to the even-numbered and odd-numbered samples of $x(n)$, respectively, that is,

$$f_1(n) = x(2n)$$

$$f_2(n) = x(2n+1), \qquad n = 0,1,\dots\dots, \frac{N}{2} - 1$$

Thus, $f_1(n)$ and $f_2(n)$ are obtained by decimating $x(n)$ by a factor of 2. Now the N-point DFT can be expressed in terms of the DFT's of the decimated sequences as follows:

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn}\ , k = 0,1,\dots.N-1$$

$$= \sum_{n\ even} x(n)W_N^{kn} + \sum_{n\ odd} x(n)W_N^{kn}$$

$$= \sum_{m=0}^{\frac{N}{2}-1} x(2m)W_N^{k2m} + \sum_{m=0}^{\frac{N}{2}-1} x(2m+1)W_N^{k(2m+1)}$$

But $W_N^2 = W_{N/2}$. With this substitution, the equation can be expressed as

$$X(k) = \sum_{m=0}^{\frac{N}{2}-1} f_1(m)W_{N/2}^{km} + W_N^k \sum_{m=0}^{\frac{N}{2}-1} f_2(m)W_{N/2}^{k(m)}$$

$$= F_1(k) + W_N^k F_2(k) \ \ k = 0,1,\dots.N-1$$

Where $F_1(k)$ and $F_2(k)$ are the $N/2$-point DFTs of the sequences $f_1(m)$ and $f_2(m)$, respectively. Since $F_1(k)$ and $F_2(k)$ are periodic, with period $N/2$, we have $F_1(k+N/2) = F_1(k)$ and $F_2(k+N/2) = F_2(k)$. In addition, the factor $W_N^{k+N/2} = -W_N^k$. Hence the equation may be expressed as

$$X(k) = F_1(k) + W_N^k F_2(k),\ k = 0,1\ldots..,N/2-1$$

$$X\left(k+\frac{N}{2}\right) = F_1(k) - W_N^k F_2(k),\ k = 0,1\ldots,\frac{N}{2}-1$$

Thus, in one iteration the computational effort is decreased by a factor of 2. Proceeding in this way recursively, the DFT of a large sequence is calculated with much less effort. The peaks in the curve are determined using efficient peak picking algorithm and displayed on the LCD screen along with the plot.

4. RESULTS DISCUSSION

4.1 PROCEDURE: The ADXL345 accelerometer is connected to the arduino board and is attached to a cantilever beam. The LCD is connected to the arduino. The arduino program starts scanning data with the push of on board button and scans upto 512 data points. The data is sent to the processor at 9600 BAUD and since the data is 24 bits, it update the processor with the Arduino's measured frequency at 9600/24 = 400 Hz. So 400 times a second the Arduino can tell the processor what frequency it measures, though, the measurement having a maximum frequency of 1600 Hz, as mentioned above. For the Arduino to simply pass the raw data to the processor at 3200 Hz, 24 bits * 3200 Hz = 76800 BAUD is minimum requirement. Hence with the arduino program of 9600 BAUD, 200 Hz maximum frequency can be successfully measured. With the 512 data points real time FFT is calculated and results are displayed on the LCD Screen.

4.2 RESULTS:

4.2.1 THEORITICAL: Dimensions of the cantilever beam used - breath: 24.77 mm; thickness: 1.41 mm and length: 297.5 mm. The beam is made of steel, hence, density = 7850 kg/m^3. According to Euler-Bernoulli theory[4] natural frequencies of a cantilever beam is given by: $w_{nf} = \alpha_n^2\sqrt{EI/mL^4}\ where\ n = 1,2,3..\infty;\ \alpha = 1.875, 4.694, 7.885..(2n-1)\pi/2$
Here $w_{nf} = 2\pi f$=circular frequency, I = 2nd moment of inertia, m= mass per unit length, L = length of the beam, E = Young modulus. Substituting, we get first natural frequency = 10Hz, second natural frequency = 63 Hz.

4.2.2 EXPERIMENTAL: From the experimental results we get the natural frequencies of the beam 10 Hz and 65 Hz, which is in accordance with the theoretical results.

5. CONCLUSION

Though the first two frequencies are successfully determined, the other natural frequencies are not reflected due to the constraints of the maximum frequency range. Some errors are also creeping in while determining the frequency values. These deficiencies are to be overcome to make it an efficient, low cost, hand-held FFT analyser for model analysis.

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