A Semi-Analytical Adaptive Sparse Surrogate based Approximation Model: Application in Non-Linear Stochastic Dynamics and Vibration Control

Tanmoy Chatterjee[a*] and Rajib Chowdhury[b]

[a] Research Scholar, Dept. of Civil Engineering, IIT Roorkee, Roorkee, India
[b] Assistant Professor, Dept. of Civil Engineering, IIT Roorkee, Roorkee, India

1. INTRODUCTION

The progressive computational effort associated with stochastic analysis of large-scale engineering systems requires efficient surrogate modeling techniques. In order to cater these demands, the following have been executed as cardinality of the present work:

- Adaptive sparse version of generalized ANOVA-decomposition (GANOVA) has been developed based on a compressive sensing technique. This feature will allow accurate recovery of solutions in a computationally efficient manner for high-dimensional problems.
- Explicit formulae have been proposed for approximating the first two statistical moments of system responses. This renders significant savings in computational effort as no further simulations are required to approximate statistical quantities of responses.
- The proposed approach (PA) has been applied to solve two problems of stochastic dynamics and vibration control. The results achieved have been validated with that of Monte Carlo simulation (MCS).

The proposed GANOVA model has been constructed by coupling conventional GANOVA [4] with polynomial chaos expansion (PCE) [2]. Considering N^{th} order ANOVA decomposition and p^{th} order PCE,

$$\hat{g}(\mathbf{x}) = g_0 + \sum_{|\mathbf{i}|=1}^{N} \sum_{|\mathbf{j_i}|=1}^{p} \alpha_{\mathbf{j_i}}^{\mathbf{i}} \psi_{\mathbf{j_i}} \tag{1}$$

Where, g_0, α, ψ are mean response, unknown coefficients and PCE bases, respectively. Then, adaptive sparsity has been achieved by utilizing compressive sensing technique based on ℓ_0-norm. Specifically, Orthogonal Matching Pursuit (OMP) algorithm [1] has been utilized for this purpose. Analytical formulae for statistical moments have been derived by utilizing few fundamental properties of PA. Next, the numerical problems have been described and results obtained by PA have been discussed briefly.

2. NUMERICAL STUDY

2.1 Base isolated single degree of freedom (SDOF) structural model

The base isolated SDOF structure is subjected to seismic ground motion, modelled as filtered white noise process [3]. Structural time period and damping, isolator time period and damping,

* Corresponding author
Tanmoy Chatterjee: E-mail: tanmoydce88@gmail.com, Telephone: 8954228997
Rajib Chowdhury: E-mail: rajibfce@iitr.ac.in

frequency and damping of filter, have been considered as stochastic parameters. The results have been presented in table 1 below.

Table 1. Comparison of two statistical moments as obtained by utilizing PA and MCS in example 2.1. n refers to number of actual function evaluations.

Approach	Response parameters			
	Maximum displacement		Maximum velocity	
	1^{st} moment	2^{nd} moment	1^{st} moment	2^{nd} moment
PA (n=500)	0.0402	0.0019	0.2100	0.0532
MCS (n=10^4)	0.0406	0.0020	0.2159	0.0575

2.2 SDOF structural model attached with tuned mass damper (TMD)

The SDOF structure attached with TMD is subjected to seismic ground motion, modelled as filtered white noise process [3]. Structural time period and damping, frequency and damping of filter, have been considered as stochastic parameters. Mass ratio, tuning ratio and TMD damping have been assumed to be 2%, 1, and 0.07, respectively. The results have been illustrated in table 2.

Table 2. Comparison of two statistical moments as obtained by utilizing PA and MCS in example 2.2. n refers to number of actual function evaluations.

Approach	Response parameters			
	Maximum displacement		Maximum velocity	
	1^{st} moment	2^{nd} moment	1^{st} moment	2^{nd} moment
PA (n=500)	0.0362	0.0015	0.3765	0.1448
MCS (n=10^4)	0.0373	0.0017	0.3726	0.1471

3. CONCLUSION

In both of the above problems, PA has achieved similar results as compared to MCS as observed in tables 1 and 2, which illustrates its approximation accuracy. It is worth mentioning that PA has utilized analytical formulae for the determination of the statistical moments and did not employ any simulations. Hence, this semi-analytical framework of PA saves significant level of computations. The study illustrates good performance of PA, making it potential for further complex applications.

REFERENCES

1. A. Doostan and H. Owhadi, "A non-adapted sparse approximation of PDEs with stochastic inputs," *J. Comput. Phys.* **230**, pp. 3015-3034, 2011.
2. B. Sudret, "Global sensitivity analysis using polynomial chaos expansions," *Reliab. Eng. Syst. Saf.* **93**, pp. 964-979, 2008.
3. BK Roy and S. Chakraborty, "Robust optimum design of base isolation system in seismic vibration control of structures under random system parameters," *Struct. Saf.* **55**, pp. 49-59, 2015.
4. T. Chatterjee and R. Chowdhury, "An efficient sparse Bayesian learning framework for stochastic response analysis," *Struct. Saf.* **68**, pp. 1-14, 2017.