Optimal path planning for fixed-wing UAVs in 3D space

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1. INTRODUCTION & OBJECTIVE

Recently, unmanned aerial vehicles (UAVs) are used in several dull, dirty and dangerous applications such as, surveillance, battle filled deployment, search and rescue etc. For almost all these missions, optimal path planning is an important problem to consider as it saves the time and fuel consumption of UAVs. The shortest path between given initial and final positions and orientations for a turn-rate constraint vehicle was first developed by Dubins [1] in 2D plane. These paths are of type CCC, CSC or any subset of these, such as CS, SC, CC and so on, where C represents the circular arc of minimum turn radius and S represents a straight line segment. Later, Sussmann [2] discussed the shortest path in 3D space which is either a helicoidal arc or CSC or CCC type path for given initial and final positions and orientations. It [2] also commented that for sufficiently large distance between two points, CSC path is optimal and for small distance, helicoidal path is optimal. But how to compute this CSC path was a difficult issue and which was addressed by Hota and Ghose [3]. This path has a potential applicability in the path planning of fixed wing UAVs with bounded turn radius flying in 3D space. Pharpatara et al. [4] discussed the 3D shortest path planning for a hypersonic glider in a heterogeneous environment. Cicibas et al. [5] presented the comparison of 3D versus 4D (three spatial dimension and time dimension) path planning of unmanned aerial vehicles and shown that the result of 4D path planning is better than 3D in complex dynamic environment. In another interesting work, Hota and Ghose [6] presented waypoint-based trajectory planning of fixed-wing MAVs in 3D space, where transition between one waypoint segment to another is executed in a smooth manner, without colliding with the obstacles and satisfying the objective of passing through a desired distance from the associated waypoint as well.

In this present work, the objective is to optimize the path length between the initial position (x_0) with given orientation (v_1) to the final position (x_f) with given orientation (v_2) in 3D space. The initial and final points are assumed to be situated sufficiently far away. The problem definition is similar to the one discussed in [3]. But in [3], for both the circular turns (initial and final), the same turning radius was assumed, which gives a longer path length compared to the path generated in this current work. This is due to the fact that in [3], the minimum turn radius was considered to be the same for all maneuvering planes for simplicity and for that the maximum value of minimum turn radius of all 2D circular maneuvers was considered. In reality, the minimum turn radius (r) of the vehicle depends on the inclination angle (ψ) of the 2D maneuver plane in 3D space. For example, the minimum turn radius in a plane which is parallel to X-Y plane is less than that of a plane parallel to X-Z plane. More specifically, the lowest value of minimum turn radius (r_h) can be achieved if the maneuver is parallel to XY plane and the

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highest value of minimum turn radius (r_v) is achieved in case of maneuvering parallel to XZ plane. For any other inclined plane the value of minimum turn radius is between these bounds, $r_h \le \mathrm{r} \le r_v$. The expression for the minimum turn radius in any plane can be obtained from analyzing Fig 1(a) for a UAV which is flying at a constant airspeed (v_a). The maximum value of load factor is assumed as a constant and is denoted by η. From the figure we get,

$$W\cos(\psi) = L\cos(\psi + \phi) \quad \text{and} \quad F_r = \frac{m v_a^2}{r} = L\sin(\psi + \phi) - W\sin(\psi) \tag{1 \& 2}$$

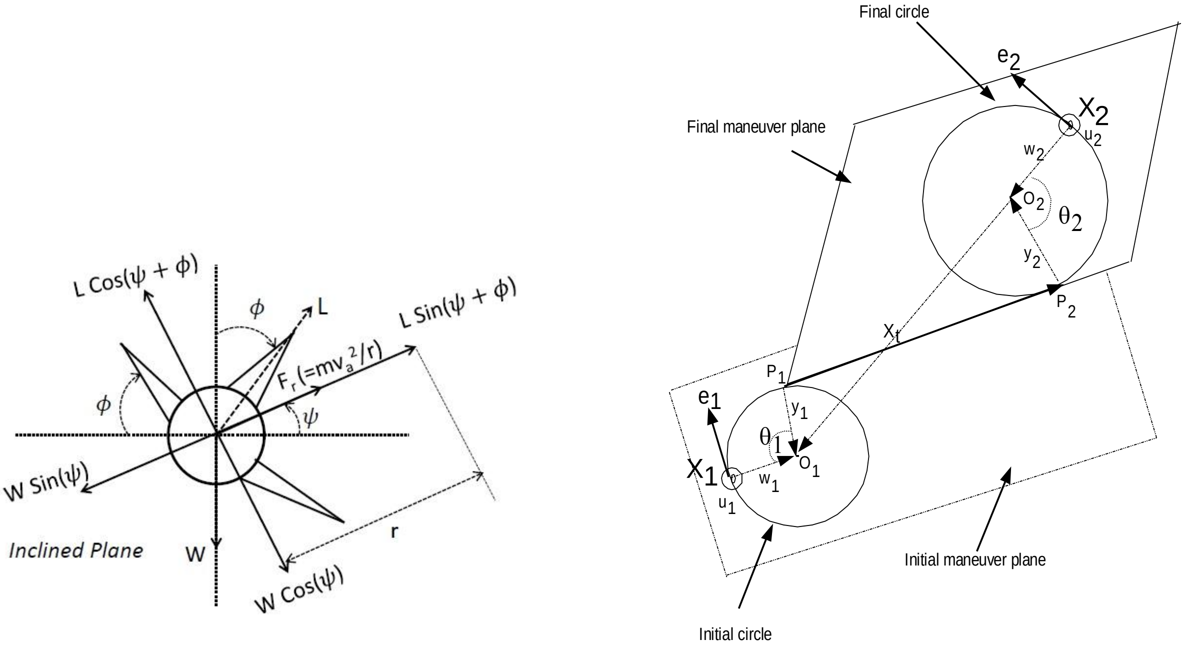
Load factor can be defined as, $\eta = L / W$ (3)

Using the above three equations we can get the value of minimum turn radius for any inclined plane as,

$$r = \frac{v_a^2}{g\left(-\sin\psi + \sqrt{(\eta^2 + \sin^2\psi - 1)}\right)} \tag{4}$$

Where ‘g’ represents the gravitational constant.

From the above discussion it is clear that if the two circular turns (initial and final circular maneuvers of a CSC path) are with different value of turn radiuses ($\le r_v$), total path length can be made smaller compared to [3]. So in this current work, the minimum turn radiuses for both initial and final maneuver planes are different (Fig. 1(b)) and which can be obtained along with the common tangent between two circular maneuvers by solving a couple of nonlinear equations. The details of which will be presented in the full length version of the current paper.



(a) **(b)**

Fig. 1: (a) Geometry for computing the minimum turn radius in inclined plane, and (b) Geometry of CSC path in 3D space for given initial and final positions and orientations.

2. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

The geometrical method is used for path planning of an aerial vehicle in 3D space with different values of minimum turn radius in different inclined planes. The method is very effective and fast enough to implement in real-time when the initial and final points are given, and they are situated sufficiently far from each other. Using the method discussed in the present work, four CSC paths can be obtained; one of them is the optimal one. This method generates the optimal path by which we can save time as well as the fuel consumption of aerial vehicles for a mission which requires a longer duration of time for its completion.

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