Spectral finite element for dynamic analysis of piezoelectric laminated composite beams

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1. INTRODUCTION & OBJECTIVE

In the recent years, the study of smart materials with piezoelectric components has attracted significant researchers due to their increasing applications in structural health monitoring, shape control, vibration suppression, noise control, etc. Piezoelectric materials are used both as actuators and sensors due to their electromechanical coupling characteristics. Several researchers have studied the modeling and analysis of the structures having piezoelectric actuators. The spectral finite element method (SFEM) is a more promising tool to investigate wave propagation in smart structures. To the best of authors' knowledge, studies on piezoelectric laminated composite beams using frequency domain SFEM has not been well addressed in the literature so far. In the present work, a spectral finite element model in frequency domain is presented for the dynamic analysis of laminated composite beams with piezoelectric actuators. The displacement field of the beam is represented by first order shear deformation theory (FSDT).

The constitutive relations for the piezoelectric smart beam are

$$\sigma_x = \hat{Q}_{11}\varepsilon_x - \hat{e}_{31}E_z, \quad \tau_{zx} = \hat{Q}_{55}\gamma_{zx}, \quad D_z = \hat{e}_{31}\varepsilon_x + \eta_{33}E_z \tag{1}$$

where σ_x, τ_{zx}, ε_x, γ_{zx}, D_z and E_z, denote the axial stress, shear stress, normal strain, shear strain, electric displacement and electric field, respectively. $\hat{Q}_{11}$, $\hat{Q}_{55}$, $\hat{e}_{31}$ and η_{33} are the reduced elastic, piezoelectric and dielectric coefficients.

The electric potential (ϕ) for the piezoelectric layer is expressed in two ways;

(i) distributed linearly through the thickness,

The electric voltage in the piezoelectric layer is expressed as $\phi(x,z) = z\phi_0(x)/h_p$ and the resulting electric field is $E_z = -\partial\phi/\partial z = -\phi_0/h_p$ where h_p is the thickness of the piezo layer.

(ii) through thickness distribution consistent with FSDT [1]

$$\phi_i(x,z) = \bar{\phi}_i(x) + (\tilde{\phi}_i(x)(z-\bar{z}_i)/h_i) - (\hat{e}_{31}h_i^2(1-4(z-\bar{z}_i)^2/h_i^2)\psi'/8\eta_{33}) \tag{2}$$

where $\bar{\phi}_i = (\phi_{i+1}+\phi_i)/2$, $\tilde{\phi}_i = \phi_{i+1}-\phi_i$, $\bar{z}_i = (z_{i+1}+z_i)/2$ and ϕ_{i+1}, ϕ_i are the electric potential at the top and bottom faces of *i*th piezo layer.

The resulting electric field is $E_z = -\partial\phi_i(x,z)/\partial z = -(\tilde{\phi}_i(x)/h_i) - (\hat{e}_{31}(z-\bar{z}_i)\psi'/\eta_{33})$.

The equations of motion and the corresponding variationally consistent boundary conditions for the beam with piezo layers are derived from the Hamilton's principle. The spectral element is derived from the exact solutions of the frequency domain governing equations, which are obtained through Fourier transformation of the time domain equations of motion using the procedure described in [2].

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2. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

The SFEM developed using (i) and (ii) for ϕ are denoted as FSDT-A and FSDT-M, respectively. These elements are employed to perform dispersion and free vibration analyses of smart composite beams. The results for natural frequencies for an asymmetric piezoelectric cantilever beam using present SFEM is compared with the FEM results [1] in Table 1. The comparison shows good agreement with the published results. Figure 1 shows the dispersion relations, i.e., the variations of the non dimensional wavenumber with the frequency, for the same asymmetric beam with and without a surface bonded PZT layer. It is observed that, at a given frequency, the wavenumbers corresponding to axial (k_1h), flexural (k_2h) and shear rotation (k_3h) modes with PZT layer is more than that of the beam without the PZT layer.

Table 1. Comparison of natural frequency of asymmetric piezoelectric cantilever beam in open circuit condition.

Thickness ratio	FSDT-M (present)	FSDT-A (present)	FSDT-M [1]	FSDT-A [1]
0.05	398.87	398.87	398.93	398.93
0.1	382.69	382.69	382.67	382.66
0.2	355.84	355.84	356.03	355.87
0.3	337.52	336.91	337.45	336.89
0.4	326.54	325.01	326.57	325.17
0.5	321.96	319.21	322.03	319.24
0.6	321.35	316.47	321.48	316.61
0.7	321.35	313.72	321.54	313.76
0.8	317.69	305.79	317.58	305.71
0.9	302.43	284.42	302.41	284.40
1.0	261.54	232.24	261.62	232.14

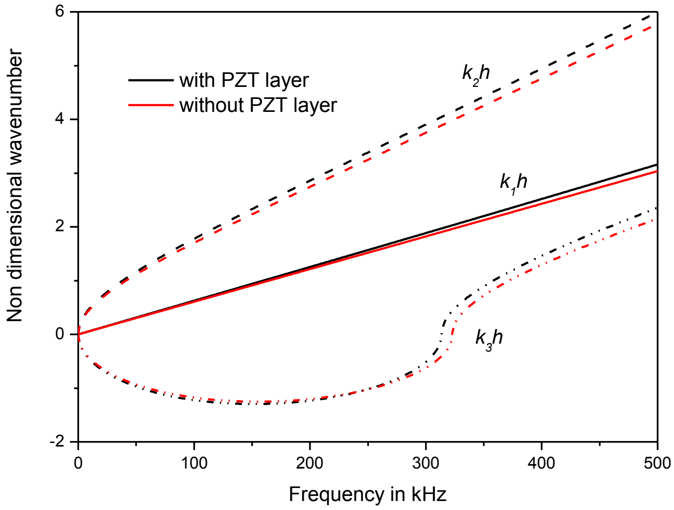

Fig. 1 Dispersion relations with and without PZT layer at top

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