HDMR Based Finite Element Model Update in Structural Damage Identification

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1. INTRODUCTION

In most of the engineering systems, much effort has been invested for developing sophisticated computer model, and to analyze the complex finite element (FE) models to predict the response of the system to the design advantage which might be obtained from modifications in the configuration of the system[1]. The FE models can be utilized to co-relate the results obtained from experimental investigations. There exists a lack of correlation of results between actual response and FE models due to the presence of certain errors in the FE models. Such errors are inherent in the model due to inappropriate boundary conditions, inaccurate material properties, discretization of continuum or a poor quality mesh, difficulty in modelling complex real life shapes etc. Thus there is a need to correct the FE model so that its behavior matches with the actual response obtained experimentally. The procedure used to update the model is called FE model updating (FEMU) [2]. In the present paper, the application of high dimensional model representation (HDMR) in FEMU and structural damage identification (SDI) in conjunction with genetic algorithm is presented with numerical examples.

2. HIGH DIMENSIONAL MODEL REPRESENTATION

The HDMR is an assumed form of mathematical expression in which the high order correlated effects of the inputs are expected to have negligible effect on the output[3]. In recent years, the application of HDMR is extended to uncertainty analysis. When the uncertainties are represented in terms of fuzzy membership functions, analysis of response of the structures is done using HDMR based response surface models[4].

The HDMR expansions are useful for the purpose of representing the outputs of a physical system when the number of input variables is large. The HDMR approximations should not be viewed as first- or second-order Taylor series expansions nor do they limit the nonlinearity of $f(\mathbf{x})$. Furthermore, the approximations contain contributions from all input variables. The HDMR expresses the output as a hierarchical correlated function expansion in terms of the input variables as:

$$f(\mathbf{x}) = f_0 + \sum_{i=1}^{N} f_i(x_i) + \sum_{1\le i\le j\le N} f_{ij}(x_i, x_j) + \sum_{1\le i\le j\le k\le N} f_{ijk}(x_i, x_j, x_k) + \ldots + f_{12\ldots N}(x_1, x_2, \ldots, x_N) \quad (1)$$

where, f_0 denotes the mean response of $f(\mathbf{x})$ which is a constant. The function $f_i(x_i)$ and $f_{ij}(x_i, x_j)$ indicates first-order and second order terms. The last term $f_{12..N}(x_1, x_2, \ldots, x_N)$ contains any

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residual dependence of all the input variables locked together in a cooperative way to influence the output. Once all component functions in Eq. (1) are determined and suitably represented, the component functions constitute the HDMR, thereby replacing the original computationally expensive method of calculating $f(\mathbf{x})$ by the computationally efficient model. To determine the component functions in Eq. (1), cut-HDMR procedure is used in approximating a univariate or a multivariate piece-wise continuous function with an equivalent continuous function. Using the cut-HDMR method, first a reference point $\mathbf{c} = \{c_1, c_2 \ldots c_N,\}$ is defined in the variable space. The expansion functions are determined by evaluating the input–output responses of the system relative to the defined reference point **c** along associated lines, planes, sub-volumes, etc. (i.e. cuts) in the input variable space.

3. DAMAGE IDENTIFICATION USING HDMR BASED MODEL UPDATING

A simulated simply supported beam without damage and with several assumed damage elements are considered[5]. The density (ρ) and elastic modulus (E) of the material of the beam are 2500 kg/m^3 and 3.2E+10N/m^2, respectively. Size of the beam is considered as 0.25m x 0.2m (Fig.1). Modal analysis is carried out by using FE analysis package for both damaged and undamaged beam. Damages are assumed by reducing the stiffness of the elastic modulus of elements 3, 8 and 10 by 20%, 50% and 30% respectively.

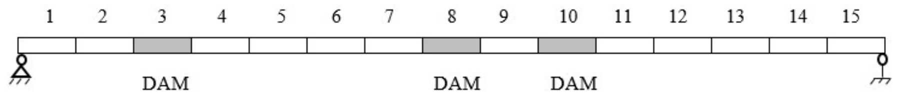

Figure. 1 A simulated simply supported beam

To develop response surface equation using HDMR, elastic modulus is considered as variable and elastic moduli of the 15 elements of the beam are considered as 15 updating parameters (x_1 to x_{15}) and first ten fundamental natural frequencies are considered as output responses. The objective is to find out the damage location and severity by updating the undamaged beam. By considering the first order approximation equation with number of variables, $N = 15$ and sample points $n = 5$. Using the above values, responses are found out for functions using finite element analysis for all the three parameters to constitute the component functions in Eq. (1). Let c_1 to c_{15} be the reference point, where output response of the system is evaluated and taken as 32 GPa. The HDMR expressions are developed for first ten natural frequencies and an objective function is then built up using the residuals between the measured (or true) responses and the predicted responses from the HDMR expressions. Based on the true results for the first ten natural frequencies the objective function can be written as follows:

$$F_{obj} = \sqrt{\begin{array}{l}(Y_1 - 8.25)^2 + (Y_2 - 34.85)^2 + (Y_3 - 74.68)^2 + (Y_4 - 135.43)^2 + (Y_5 - 141.06)^2 + \\ (Y_6 - 204.69)^2 + (Y_7 - 298.58)^2 + (Y_8 - 386.98)^2 + (Y_9 - 417.32)^2 + (Y_{10} - 494.62)^2\end{array}} \quad (2)$$

where, Y_1, to Y_{10} denote the first ten natural frequencies of the beam obtained using HDMR. Objective function developed in Eq. (2) is optimized using the GA. Table 1 shows the frequencies of undamaged and damaged beam where frequencies of damaged beam are taken as true values. Figure 2 indicates the detected damage pattern by model updating using HDMR and it can be seen that the damage % for element 3, 8 and 10 are found to be 31% 45% and 34%, and also in other elements where damage is detected are less than 13%.

Table 1 Frequencies of simulated beam

Mode	Undamaged beam (Hz)	Damaged beam (Hz)
1	9.00	8.26
2	35.86	34.84
3	80.13	74.69
4	141.03	135.43
5	149.00	141.06
6	217.39	204.69
7	307.52	298.58
8	409.13	386.98
9	445.38	417.32
10	519.16	494.62

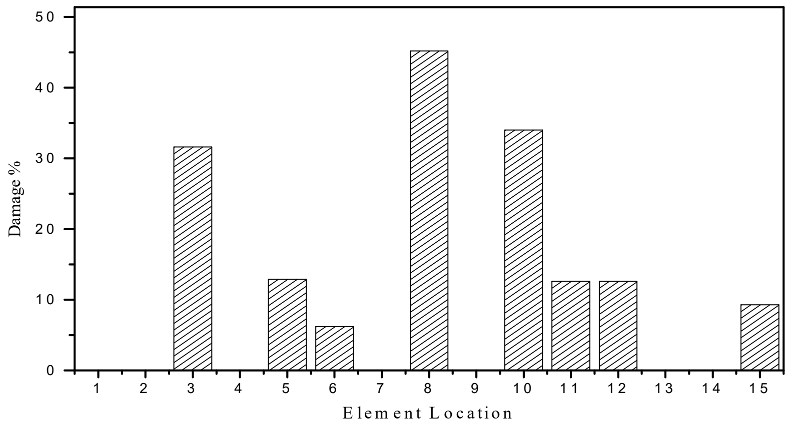

Figure. 2 Location and severity of damage in simulated beam by model updating using HDMR

4. CONCLUSIONS

In this paper, a first order HDMR expressions are developed for response equations, and an objective function is built up using the residuals between measured and predicted responses from the developed HMDR equations, and updated parameters are obtained using the GA. From the numerical examples presented, damage patterns are found out by model updating using HDMR. Further the accuracy of the results can be significantly improved by employing the second order HDMR, but with slightly increased computational effort.

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